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A Design Method of Control-Theoretic B-spline Curves for Modeling Dynamic Font-based Hairy-brush Characters

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Abstract

This paper presents a design method of the so-called “control-theoretic B-spline curves” for modeling hairy-brush characters, as shown in Japanese calligraphy. The curves are constituted by employing normalized, uniform B-splines as basis functions. Then, we formulate the curve-design problem as an optimal control problem, which minimizes an input energy of trajectory in the stroke of characters. A modeling example of characters is included.

Keywords : Curves, B-splines, Control-theoretic Approach, Modeling, Characters.

1. Introduction

The primary purpose of this study is to develop a design method of the so-called “curves”. A problem of designing curves arises in many industrial applications such as robotic path following problem⁽¹⁾ and image registration⁽²⁾, etc. Therein, the spline curves have been frequently used. In particular, Egerstedt and Martin have developed the theory of such splines from the viewpoint of optimal control theory⁽³⁾, which has been often called as “dynamic splines”. Also, such an approach led an analysis on B-spline curves⁽⁴⁾, which is referred to as “control-theoretic B-splines” in this paper.

On the other hand, as an application using spline curves, we have focused on the problem of constructing and reconstructing characters based on the so-called “dynamic font” method⁽⁵⁾. The dynamic font method has been developed based on the writing process by humans. Therein, characters are generated by moving a writing device on a writing plane continuously in both time and space. This method is quite different from the ordinary design methods such as dot matrix, outline vector, and skeleton vector methods, etc. ⁽⁶⁾, in which the characters are generated as planar static patterns. Moreover, this dynamic font method is powerful, particularly when we want to generate and manipulate characters in Japanese calligraphy, where the thickness of the stroke is important. However, in these character designs, the hairy-brush ink effect as seen in real Japanese calligraphy has never been considered.

In order to generate such a hairy-brush ink effect on characters, one of the typical approaches may be a “model-based approach”. As an example of using a model-based approach, Wong and Ip⁽⁷⁾ have developed a writing device model that can create characters with hairy-brush ink effects by using a parameterized model which captures “the writing brush 3D geometry”, “the brush hair properties”, and “the variations of ink deposition along a stroke trajectory”. However, this approach may face some difficulties in

optimally setting of the parameters, which are required for finely generating hairy-brush ink effect. In addition, the computational time is excessive as the fineness of hairy-brush ink becomes high.

An alternative approach may be to introduce a deep-learning approach. Recently, we have developed a method for generating hairy-brush ink using the so-called “Cycle-GAN”⁽⁸⁾. That is, the neural network-based model is developed to directly generate hairy-brush images from some standard character font images directly. Then, the fineness of hairy-brush ink may be achieved independently with the low computational time. These characters are generally treated as planar static patterns. But, Xie and Fujioka et al. ⁽⁹⁾ have very recently presented that the typefaces of pre-designed dynamic font characters with one-pixel thickness can be successfully transformed to hairy-brush ones by the deep learning approach. Moreover, we have presented the hairy-brush ink data is scanned and stored as time-series data based on the trajectory of a pre-designed dynamic font.

Therefore, what we want to do here is to develop a method for designing curves from a set of scanned and stored brush ink data for modeling dynamic font-based hairy characters. In particular, we develop such a method by employing the control-theoretic function approximation theory. Only such modeled characters are taken by using our proposed method, it will enable us to readily reconstruct characters, e.g., resizing, rotation, and concatenation, etc. by utilizing the advantages of dynamic font method⁽⁵⁾.

The paper is organized as follows. In Section 2, we briefly present the problem statement. Then in Section 3, we present a method scheme for designing B-spline curves based on the control-theoretic B-spline smoothing. Also, a modeled example is included. Concluding remarks are given in Section 4.

2. Problem Statement

First, we present the problem statement. Now that we are given some hairy-brush characters as shown in Fig. 1 (a), where such characters are generated using the deep learning approach⁽⁸⁾ from the pre-designed dynamic font pre-designed dynamic font characters with one-pixel thickness characters with one-pixel thickness in Fig. 1 (b). Note here that the pre-designed characters are generated based on dynamic font. Hence the characters are generated as a trajectory $x_d(t) = [X_d(t) Y_d(t)]^T \in \mathbf{R}^2, t \in [t_0, t_m]$ defined by

$$x_d(t) = \sum_{i=-3}^{m-1} p_i B_3(\alpha(t - t_i)) \quad (1)$$

where $B_3(\cdot)$ denotes a normalized, uniform cubic B-splines⁽⁵⁾, m is an integer that determines the time interval of $x_d(t)$, p_i 's are two-dimensional weighting vector called as ‘‘control point’’, and $\alpha (> 0)$ is a scalar for scaling the interval between equally-spaced knot points t_i with

$$t_{i+1} - t_i = \frac{1}{\alpha} \quad (2)$$

Moreover, we may be able to regard the pre-designed characters as the skeleton of the hairy-brush characters from Fig. 1. Then, using the method proposed by Xie and Fujioka et al. ⁽⁹⁾, it can be shown that the hairy-brush ink data are scanned and stored along with the pre-designing characters, i.e. $x_d(t)$ in (1) as follows.

Now, let $n(t) \in \mathbf{R}^2$ be a normal unit vector to the motion of pre-designed characters on O-XY plane, say $x_d(t)$ in (1), we can compute $n(t)$ as

$$n(t) = \frac{1}{\sqrt{(\dot{X}_d(t))^2 + (\dot{Y}_d(t))^2}} [-Y_d(t) \quad X_d(t)]^T \quad (3)$$

where $\dot{X}_d(t)$ and $\dot{Y}_d(t)$ denote $dX_d(t)/dt$ and $dY_d(t)/dt$, respectively. We then obtain a set of hairy-brush ink data by scanning the transformed characters to the normal direction $n(s_i)$ from $x_d(s_i)$ at a sampling time $s_i \in [t_0, t_m]$ for $i = 1, 2, \dots, N$ as follows. Let $x_d^k(s_i) = [X_d^k(s_i) \quad Y_d^k(s_i)]^T \in \mathbf{R}^2$ be a point given by

$$x_d^k(s_i) = x_d(s_i) + ak \cdot n(s_i) \quad (4)$$

with $a (> 0) \in \mathbf{R}^2$ and $v^k(s_i) \in \mathbf{N}$ be the gray-scale value on $x_d^k(s_i)$ at a sampling time s_i . Also, $k \in [k_L, k_U] \subset \mathbf{Z}$ is an index of scanning line given by $x_d^k(s_i)$, where k_L and k_U are set so that the region between $x_d^{k_L}(t)$ and $x_d^{k_U}(t)$ cover the whole



(a) hairy-brush character (b) pre-designed character

Fig. 1, An example of hairy-brush characters and pre-designed characters.

character. We thus measure and store a set of hairy-brush ink data for each k , denoted as D_k , as

$$D_k = \{(s_i, d_i^k) : d_i^k = \left[\begin{matrix} x_d^k(s_i) \\ v^k(s_i) \end{matrix} \right]^T \in \mathbf{R}^3, s_i \in [t_0, t_m], i = 1, 2, \dots, N\}. \quad (5)$$

Then, we consider the following problem of modeling characters by employing the dynamic font method.

Problem 1: Suppose that a set of hairy-brush ink data for each k , i.e. D_k in (5) is given and that an appropriate α and m in (1) are chosen. Construct the corresponding hairy-brush ink motion $x_k(t) = [X_k(t) \quad Y_k(t) \quad v_k(t)]^T \in \mathbf{R}^3, t \in [t_0, t_m]$ for index k , which can be represented with the same form as (1), i.e.

$$x_k(t) = \sum_{i=-3}^{m-1} p_i^k B_3(\alpha(t - t_i)) \quad (6)$$

where $p_i^k \in \mathbf{R}^3$ are control points.

Equivalently, letting \mathcal{M}_k be a sequence of control polygon, referred as ‘‘control polygon’’, given as

$$\mathcal{M}_k = p_{-3}, p_{-2}, \dots, p_{m-1}. \quad (7)$$

We can regard Problem 1 to find \mathcal{M}_k given as

$$\mathcal{M} = \cup_{\{k=k_L\}}^{\{k=k_U\}} \mathcal{M}_k \quad (8)$$

which is a formal representation of the hairy-brush characters. Also, \cup denotes the multiset union. Then, hairy-brush characters are generated by filling the points $x_k(t)$ with the gray-scale value $v_k(t)$ for $\forall t \in [t_0, t_m]$ and $\forall k \in [k_L, k_U]$, which are determined from \mathcal{M} .

3. Control-Theoretic B-spline Curves

For solving Problem 1 in the previous section, we are now in the position to present the control-theoretic B-spline curves. For the sake of simplicity, we assume that each element in $x_k(t) \in \mathbf{R}^3$ is modeled independently. Thus, p_i^k and d_i^k are assumed to be one-dimensional. Also, we set the parameter α as $\alpha = 1$.

Let us consider a linear system

$$\frac{d^3}{dt^3} x_k(t) = u_k, \quad x_k(-3) = \dot{x}_k(-3) = \ddot{x}_k(-3) = 0 \quad (9)$$

and a restricted set of controls

$$U_k = \left\{ u_k(t) : u_k(t) = \sum_{i=-3}^{m-1} p_i^k \frac{d^3}{dt^3} B_3(t-i), p_i^k \in \mathbf{R} \right\}. \quad (10)$$

Here, we assume that m is $m \geq 0$ and the first and last three coefficients of the control points are prescribed as

$$p_k^I = \begin{bmatrix} p_{-3}^k \\ p_{-2}^k \\ p_{-1}^k \end{bmatrix} = d_1^k \cdot \mathbf{1}_3, \quad p_k^F = \begin{bmatrix} p_{m-3}^k \\ p_{m-2}^k \\ p_{m-1}^k \end{bmatrix} = d_N^k \cdot \mathbf{1}_3 \quad (11)$$

with $\mathbf{1}_k = [1 \ 1 \ 1]^T \in \mathbf{R}^3$. This implies that $x_k(t)$ in (6) coincides with the data and is stationary on starting and ending. Then, we consider the following problem, in which an approximation problem is formulated as the following control problem.

Problem 2: Find an optimal input $u_k(t)$ and solution $x_k(t)$ such that

$$\min_{u_k \in U_k} J_k(u_k) \quad (12)$$

where $J_k(u_k)$ is given as

$$J_k(u_k) = \lambda \int_{-\infty}^{+\infty} (u_k(t))^2 dt + \sum_{i=1}^N (x_k(s_i) - d_i^k)^2 \quad (13)$$

with $\lambda > 0$ and $0 \leq w_i \leq 1, \forall i = 1, 2, \dots, N$.

The optimal solution of this problem can be obtained as follows. Here, let $p_k \in \mathbf{R}^M$ ($M = m + 3$) and $b_3(t) \in \mathbf{R}^M$ be

$$p_k = [p_{-3}^k \ p_{-2}^k \ \dots \ p_{m-1}^k]^T \quad (14)$$

$$b_3(t) = [B_3(t+3) \ B_3(t+2) \ \dots \ B_3(t-(m-1))]^T \quad (15)$$

Then, $x_k(t)$ in (6) can be expressed as

$$x_k(t) = p_k^T b_3(t). \quad (16)$$

Then, the cost $J_k(u_k)$ in (13) can be expressed as

$$J_k(u_k) = \lambda \ p_k^T Q_M p_k + (B p_k - d_k)^T W (B p_k - d_k). \quad (17)$$

Here, in (17), the matrix $Q_M \in \mathbf{R}^{M \times M}$ is a Gramian defined by

$$Q_M = \int_{-\infty}^{+\infty} b_3(t) (b_3(t))^T dt \quad (18)$$

and it can be computed explicitly using B-spline functions⁵. Also, the matrices $B \in \mathbf{R}^{N \times M}$, $W \in \mathbf{R}^{N \times N}$ and the vector $d_k \in \mathbf{R}^N$ are given as

$$\begin{aligned} B &= [b_3(s_1) \ b_3(s_2) \ \dots \ b_3(s_N)]^T \\ W &= \text{diag}\{w_1, w_2, \dots, w_N\} \\ d_k &= [d_1^k \ d_2^k \ \dots \ d_N^k]^T. \end{aligned} \quad (19)$$

We here notice that $J_k(u_k)$ in (17) is to be minimized with respect with \hat{p}_k defined by

$$\hat{p}_k = [p_0^k \ p_1^k \ \dots \ p_{m-4}^k]^T \quad (20)$$

since the first and last three coefficients of the control points p_k^I and p_k^F in (11) are prescribed. For this purpose, let Q_M in (18) be decomposed as

$$Q_M = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{12}^T & Q_{M-6} & Q_{23} \\ Q_{13}^T & Q_{23}^T & Q_{33} \end{bmatrix} \quad (21)$$

with $Q_{11}, Q_{13}, Q_{33} \in \mathbf{R}^{3 \times 3}, Q_{12} \in \mathbf{R}^{3 \times (M-6)}, Q_{23} \in \mathbf{R}^{(M-6) \times 3}$. Also, $Q_{M-6} \in \mathbf{R}^{(M-6) \times (M-6)}$ is the principal minor of Q_M of size $M-6$, thus Q_{M-6} is positive-definite, i.e. $Q_{M-6} > 0$. Then, the first term of (17), i.e. $p_k^T Q_M p_k$ can be written as

$$p_k^T Q_M p_k = \hat{p}_k^T Q_{M-6} \hat{p}_k + 2(Q_{12}^T p_k^I + Q_{23} p_k^F)^T \hat{p}_k + \text{const}. \quad (22)$$

Similarly to (21), let the matrix B in (19) be decomposed as

$$B = [B_I \quad \hat{B} \quad B_F] \quad (23)$$

with $B_I, B_F \in \mathbf{R}^{N \times 3}$ and $\hat{B} \in \mathbf{R}^{N \times (M-3)}$. Then, the second term of (17), i.e. $(Bp_k - d_k)^T W (Bp_k - d_k)$ is expressed as

$$\begin{aligned} (Bp_k - d_k)^T W (Bp_k - d_k) &= \hat{p}_k^T \hat{B}^T W \hat{B} \hat{p}_k \\ &+ 2(B_I p_k^I + B_F p_k^F - d_k)^T W \hat{B} \hat{p}_k + \text{const.} \end{aligned} \quad (24)$$

From (22) and (24), the cost $J_k(u_k)$ can be expressed in terms of \hat{p}_k as $\bar{J}_k(\hat{p}_k)$,

$$\begin{aligned} \bar{J}_k(\hat{p}_k) &= \hat{p}_k^T (\lambda Q_{M-6} + \hat{B}^T W \hat{B}) \hat{p}_k \\ &+ 2(\lambda(Q_{12}^T p_k^I + Q_{23} p_k^F) + \hat{B}^T W (B_I p_k^I + B_F p_k^F - d_k))^T \hat{p}_k \\ &+ \text{const.} \end{aligned} \quad (25)$$

Taking the derivative of $\bar{J}_k(\hat{p}_k)$ with respect to \hat{p}_k and setting it to zero, then we get

$$\begin{aligned} (\lambda Q_{M-6} + \hat{B}^T W \hat{B}) \hat{p}_k &= \hat{B}^T W (d_k - B_I p_k^I - B_F p_k^F) \\ &- \lambda(Q_{12}^T p_k^I + Q_{23} p_k^F). \end{aligned} \quad (26)$$

Since it holds that $\lambda Q_{M-6} + \hat{B}^T W \hat{B} > 0$, the solution \hat{p}_k can be obtained uniquely as

$$\begin{aligned} \hat{p}_k &= (\lambda Q_{M-6} + \hat{B}^T W \hat{B})^{-1} [\hat{B}^T W (d_k - B_I p_k^I - B_F p_k^F) \\ &- \lambda(Q_{12}^T p_k^I + Q_{23} p_k^F)]. \end{aligned} \quad (27)$$

Thus, there always exists a unique optimal solution to Problem 2, and the optimal control $u_k^*(t)$ and the optimal solution $x_k^*(t)$ are given by

$$u_k^*(t) = \sum_{i=-3}^{m-1} p_i^{k*} \frac{d^3}{dt^3} B_3(t-i) \quad (28)$$

$$x_k^*(t) = \sum_{i=-3}^{m-1} p_i^{k*} B_3(t-i). \quad (29)$$

Here, the first and last three coefficients of the control points p_i^{k*} are given from (11) as

$$\begin{bmatrix} p_{-3}^{k*} \\ p_{-2}^{k*} \\ p_{-1}^{k*} \end{bmatrix} = d_1^k \cdot \mathbf{1}_3, \quad \begin{bmatrix} p_{m-3}^{k*} \\ p_{m-2}^{k*} \\ p_{m-1}^{k*} \end{bmatrix} = d_N^k \cdot \mathbf{1}_3 \quad (30)$$

Also, other control points, i.e. p_i^{k*} for $i = 0, 1, \dots, m-4$ are given by (27).

By computing $p_k^{\wedge \star}$ for $\forall k \in [k_L, k_U]$, we then get \mathcal{M} in(8) as a model of hairy-brush dynamic font-based characters. An

example of modeled characters are illustrated in Fig. 2.



Fig. 2, An example of modeled characters.

4. Concluding Remarks

In this paper, we presented a design method of “curves” for modeling the dynamic font-based hairy-brush characters. The curves are constituted by employing normalized, uniform B-splines as basis functions. In particular, we here formulated the curve-design problem as an optimal function-approximation problem. Then, a central issue was to determine control points for generating optimal input and solution. The derivation and model example were also shown. As future work, it remains to develop some operations to generate words and sentences in a systematic way based on the dynamic font method.

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