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Citation	福岡工業大学研究論集 第43巻2号(通巻66号) P103-P109
Issue Date	2011-2
URI	http://hdl.handle.net/11478/1306
Right	
Type	Departmental Bulletin Paper
Textversion	Publisher

Fukuoka Institute of Technology

On Asymptotic Behavior of Electric Resistivity at Infinitesimal Electric Current in Glassy State of Superconducting Quantized Fluxoids I. Basic Formulations

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Abstract

The purpose of the present study is to answer the question: Does the electric resistivity, ρ , approach zero as the applied electric current approaches zero, in the glassy state of the quantized fluxoids in a type 2 superconductor? The best method for answering the above question by experimental studies may be to measure the relaxation process of the induced electric field, E , during ultra-long time in the concerning material. This kind of measurements are, however, very hard to perform, and hence, we tried to answer the above question by a theoretical approach. In this paper, we propose the starting basic equations to investigate the present problem by a theoretical analysis or by numerical calculations, because we have at present no established reliable theoretical formulation for describing the ultra-long time relaxation process in the glassy state.

Keywords: *glass, high-temperature superconductor, quantized fluxoids, electro-magnetic property*

1. Introduction

For the power applications of the superconductors, the type 2 superconducting materials containing a remarkable amount of inhomogeneous substances, called the pinning centers, are usually used. When the magnetic field, higher than the lower critical field, H_{c1} , is applied to a sample of this kind of materials, the sample becomes the mixed state, where the quantized fluxoids pass through the sample. For a weakly pinning sample, that contains, *e.g.*, randomly distributing point pins, the quantized fluxoids are in the glassy state at the temperatures lower than so-called the glass-liquid transition temperature [1]. When the electric current with the current density, J , is applied to the sample in this state, the electric field, E , is induced due to the motions of the quantized fluxoids.

Fisher et al. [1] predicted theoretically that the electric resistivity, $\rho \equiv E/J$, tends to $\rho \rightarrow 0$ in the limit of $J \rightarrow 0$ in the glassy state of the quantized fluxoids.

In other words, they insisted that the glassy state of superconducting quantized fluxoids becomes “the superconducting state with $\rho \rightarrow 0$ ” in the limit of $J \rightarrow 0$.

Their prediction [1] seems, however, to be at variance with the existing common sense for the glassy state: In the glassy state of any kind of particles, the particles are making thermally fluctuating motions at a finite temperature. In the presence of the electric current with current density, J , having a finite value, therefore, a finite value of the electric field, E , is induced, and hence the glassy state has been regarded as a kind of the dissipative state with a finite energy consumption. Since we cannot find any existing example for any dissipative state, where ρ approaches zero as J approaches zero, while E itself approaches zero as J approaches zero, then their prediction should be re-examined very carefully.

For this purpose, it may be instructive to mention briefly the background of their theoretical prediction [1].

The glassy state of quantized fluxoids is a very complicated system, where the assembly of fluxoids, in which fluxoids are interacting with each other, is trapped by the pinning potential resulting from the interaction between the assembly of fluxoids and the assembly of pinning centers. For the semi-quantitative explanation of the observed electro-magnetic properties in this kind of system, so-called the “collective theory” [2] has been used, while the phenomenological investigations based on the models [3] for the detailed behaviors of the assembly of pinning centers

have been used to explain quantitatively the observed electro-magnetic behavior [4,5].

Fisher et al. [1] at first predicted theoretically with the aid of the collective theory [2,6] that so-called the glass-liquid transition will appear as the temperature is raised. They also predicted that the observed characteristics in the vicinity of this kind of transition are very similar to those for the second order transition, and hence the observed E vs. J curves in the glassy state in the vicinity of the transition temperature should be scaled to a single master curve. These theoretical predictions have been confirmed by a tremendous amount of experimental works.

In the derivation of their scaling law for the E vs. J curves, Fisher et al. [1] assumed that the “collective pinning potential” for the assembly of fluxoids, $U_{p-f}(J)$, resulting from the collective interaction between the assembly of fluxoids and the assembly of pinning centers, is given by [6]

$$U_{p-f}(J) = U_{p-f}(J_c) \left[1 - \left(\frac{J_c}{J} \right)^\mu \right], \quad (1.1)$$

where J_c that is called the critical current density and the numerical index, μ , that takes a value in the range of $0 < \mu \leq 1$ are the parameters characterizing the collective pinning potential. According to (1.1), the collective pinning potential becomes infinitely deep in the limit of $J \rightarrow 0$.

It is to be mentioned that two types of motions of the assembly of fluxoids can appear in the glassy state of the fluxoids: When the driving force, which is given by the Lorentz-force type expression as $F_L = J \times B$ [2] for the unit volume of the fluxoid assembly with the flux density of B , becomes large enough so that the fluxoid assembly can move across the pinning potential, then the electric field, E_{ff} , that is called the flux-flow electric field, is induced by this kind of motion of fluxoid assembly. When the driving force is too small to result in the above-mentioned motion of the fluxoid assembly across the pinning potential, on the other hand, the fluxoid assembly remains inside the valley of the pinning potential, and hence the flux-flow electric field, E_{ff} , does not appear. At a finite temperature, however, the fluxoid assembly is making thermal motion inside the valley of the collective pinning potential. Taking account of this kind of thermal motion, too, the fluxoids assembly can move to the direction of the driving force across the pinning potential by thermal hopping after a long time. The electric field induced by the thermal hopping of the fluxoid assembly is called the flux-creep electric field, E_{fc} . When the value of J becomes much smaller than the critical current density, J_c , therefore, only E_{fc} is observed.

Since the flux-creep electric field, E_{fc} can be estimated theoretically [7] with the aid of the well-known rate theory,

so far as the expression for the pinning potential is given explicitly, it is an easy matter to show that the electric resistivity, ρ , tends to $\rho \rightarrow 0$ in the limit of, $J \rightarrow 0$ so far as the expression for the collective pinning potential is given by (1.1).

The above fact clearly indicates that the validity of the above-mentioned theoretical prediction by Fisher et al. [1] on the asymptotic behavior of ρ in the limit of $J \rightarrow 0$ depends simply on the fact that (1.1) is valid even at $J \rightarrow 0$ or not.

Let us mention two comments on this point.

First, the target of the existing theoretical investigations resulting in the collective theory [2,6] has been confined mainly to the range of the value of J near J_c , and hence the applicability of (1.1) for the smaller values of J , at which only E_{fc} is observed, should be re-examined very carefully by experimental measurements. From this point of view, we can find only a single paper [8], in which they reported the observed result that the scaling law predicted by Fisher et al. [1] by using (1.1) breaks only for very small values of the electric current density, J .

Secondly, it is to be emphasized that the theory of Fisher et al. [1] cannot explain quantitatively the observed E vs. J curves in the glassy state of quantized fluxoids, but can only describe the shape of the scaled master curve. Yamafuji et al. [5], on the other hand, showed with the aid of a phenomenological model [3] on the pinning characteristics that the observed E vs. J curves and also their temperature dependence, reported by many papers [3,4] including the paper [8], can be explained quantitatively. According to the theory [5], the corresponding expression to (1.1) for the pinning potential of the assembly of fluxoids for the flux-creep should be given by

$$U_{p-f}(J) = U_{p-f}(J_c) \left[1 - \left(\frac{J_c + \delta J_c}{J + \delta J_c} \right)^\mu \right], \quad (1.2)$$

for satisfying the requirement that the pinning potential should decrease as the temperature is raised. It is to be noted that the value of δJ_c appearing in their theory [5] is quite small, but is surely measurable.

The above comments indicate that we can at least find an existing experimental result [8] and also a theoretical investigation [5], those are against to the theoretical prediction by Fisher et al. [1], and hence the validity of the latter prediction [1] should be re-examined from a more basic theoretical background than the existing collective theory and also the existing phenomenological pinning models.

The present theoretical investigation for this purpose is expected to give a useful contribution not only on the establishment of the basic physical concept on the glassy state of quantized fluxoids, but also on the theoretical

formulation for the ultra-long time relaxation process in the glass materials.

2 . Starting equations

2.1 Langevin equations for the motion of fluxoids

The expression (1.1) for the “collective pinning potential” for the assembly of fluxoids resulted from a theoretical investigation on the fluxoid assembly that interacts with the assembly of pinning centers. In the present study, therefore, let us start from a more basic situation, in which the target of the investigation is not the motion of the fluxoid assembly, but is the motion of each single fluxoid that is interacting with other fluxoids and also with each pinning center in the assembly of pinning centers. Furthermore, the thermally fluctuating vibration of each fluxoid should also be taken into account explicitly.

When each fluxoid moves to the direction of the driving force, it encounters each pinning center successively, and suffers the pinning potential of each pinning center for a single fluxoid. It is to be noted that the pinning potential of each pinning center for a single fluxoid has been known to have some stochastically describable property that is differing from the collective pinning potential. For a weekly pinning sample, for which Fisher et al. [1] investigated, both of the position of each pinning center and the value of the strength of each pinning force are assumed [1] to be distributing randomly within respective range. Then the pinning potential of each pinning center for single fluxoid should also have the stochastic nature resulting from the above-mentioned stochastic properties of the pinning centers.

For investigating theoretically a system having some stochastic nature, it is a usual way to start from the Langevin equation. As the starting equation in this paper, therefore, let us choose a set of the Langevin equations, each of which describes the motion of each fluxoid.

It is to be emphasized that the present Langevin equations contain three kinds of stochastic characteristics, that is, for the thermal fluctuating force, for the position of the pinning centers, and also for the strength of the pinning force. However, we have at present no theoretical method to solve exactly this kind of set of three-dimensional Langevin equations. To make the situation slightly simpler, let us start from a set of the one-dimensional Langevin equations, because the assembly of fluxoids moves one-dimensionally to the direction of the driving force. The propriety of the present simplification can be checked by comparing the numerically solved results with the observed results for

relatively large values of the current density, J , for which the scaling law predicted by Fisher et al. [1] has been confirmed by many measurements.

Let us assume that the concerning system is composed of the one-dimensional array of N_f fluxoids and also the one-dimensional array of N_p pinning centers. Then let us denote the position of the p -th pinning center by $X_p = X_p(X)$, where X is the spatial coordinate of the concerning type 2 superconducting sample with the range of $0 < X < \infty$. We also denote the position of the j -th fluxoid by $X_j = X_j(t; \{X_p\})$, where t is the time with the range of $0 \leq t < \infty$.

The initial position of X_j on the sample before the electric current is applied at $t=0$ is restricted within the range of X given by $(j-1/2)a \leq X_j \leq (j+1/2)a$, where a is the average interval between successive fluxoids.

Then the Langevin equation for the j -th fluxoid can be written in the form of the force balance equation as

$$\Phi_0 J = \eta \frac{\partial X_j}{\partial t} - \frac{\partial}{\partial X_j} U_{\text{ela}}(\{X_j\}) - \frac{\partial}{\partial X_j} U_{\text{pin}}(\{X_j\}, \{X_p\}) - \Xi_j(t). \quad (2.1)$$

In equation (2.1), the left-hand-side term is the driving force acting on each fluxoid when the electric current with the current density, J , is applied to the sample, where Φ_0 is the flux quantum. On the other hand, the resistive forces against the driving force acting on the j -th fluxoid are listed in the right-hand side: The first term gives the viscous-drag force, where η is the viscosity coefficient for a single fluxoid, the second term gives the elastic force due to the attractive interaction with neighboring fluxoids, and the third term gives the pinning force.

In addition, $\Xi_j(t)$ is the thermal fluctuation force acting on the j -th fluxoid, where the stochastic nature of $\Xi_j(t)$ is characterized by the following expressions:

$$\langle \Xi_j(t) \rangle = 0, \quad (2.2a)$$

$$\langle \Xi_j(t) \Xi_i(t') \rangle = 2\eta k_B T \delta_{ji} \delta(t-t'). \quad (2.2b)$$

In equation (2.2b), T is the temperature, k_B is the Boltzmann constant, δ_{ji} is Kronecker's delta, $\delta(t-t')$ is the delta function, and $\langle \Xi(t) \rangle$ represents the average of $\Xi(t)$ in the thermally equilibrium state.

As for the elastic force, to adopt a Hook' spring-type force may be a good approximation, because the deviation of the interval between successive fluxoids, $(X_j - X_{j-1})$, from the average interval, a , is known to be small compared with a in the glassy state of the fluoids. Then the elastic-force potential can be approximated by the following expression:

$$U_{\text{ela}}(\{X_j\}) = \sum_j \frac{K}{2} (X_j - X_{j-1} - a)^2, \quad (2.3)$$

where K is the spring constant.

2.2 Expression for pinning force

The most severe hazard for solving the above-mentioned set of Langevin equations is the existence of the pinning force having the doubly stochastic characteristics. We can find no existing work to succeed to solve exactly this type of Langevin equations.

It is to be mentioned that any theoretical prediction breaks when a single experimental example, that is contradicts the prediction, is proposed, so far as the proposed result is agreed upon by everyone. From this point of view, we have only to choose a specified set of pinning forces enough for realizing a glassy state of fluxoids.

Since the expression for the pinning forces is better to be as simple as possible for solving the equations, let us choose a simple case for the distribution of the position of fluoids, where only a single pinning center exists in each equilibrium interval of the successive fluxoids in the absence of the pinning centers.

Under the present simplification, the position of the p -th pinning center, $X_p(X)$, is given by

$$X_p(X) = pa + \Delta X_p(X); \quad p=1, 2, \dots, N_p (= N_t), \quad (2.4a)$$

where the values of ΔX_p are randomly distributed inside the following range on the sample given by

$$\ell_L \frac{a}{2} \leq \left| \frac{a}{2} - \Delta X_p \right| \leq (1 - \ell_L) \frac{a}{2}, \quad (2.4b)$$

with Lindemann's criterion constant for the glassy state [8], denoted by $2\ell_L$.

In the way of the movement of each fluxoid to the direction of the driving force, the concerning fluxoid encounters the next pinning center successively. Such a situation can be formally expressed by

$$U_{\text{pin}}(\{X_j\}) = \sum_{j=1}^{j=N_t} U_j(X_j; \{X_p\});$$

$$U_j(X_j; \{X_p\}) = \sum_{p=1}^{p=N_p} \Delta(X_j; X_p) U_{pj}(X_j; X_p), \quad (2.5a)$$

where $\Delta(X_j; X_p)$ is a kind of box function defined by

$$\Delta(X_j; X_p) = 1; \quad pa \leq X_j \leq (p+1)a$$

$$= 0; \quad \textit{otherwise}. \quad (2.5b)$$

In equation (2.5a), $U_{pj}(X_j; X_p)$, which is defined only inside the region of $pa \leq X_j \leq (p+1)a$, is the pinning potential of the p -th pinning center that affects the j -th fluxoid.

A reasonable requirement on the pinning potential is that all the values of the potential and its spatial derivative and also the second spatial derivative are continuous inside the above region, and also become zero at the both boundaries of this region.

Let us propose an example of the pinning potential

satisfying the above requirement as

$$U_{pj}(X_j; X_p) = -U_p \left(\frac{X_j - pa}{\Delta X_p} \right)^4 \left[\frac{a - (X_j - pa)}{a - \Delta X_p} \right]^4$$

$$\times \left[\frac{3\Delta X_p (a - \Delta X_p)^2}{\Delta X_p (a - (X_j - pa))^2 + 2(a - \Delta X_p)^2 (X_j - pa)} \right]^6. \quad (2.5c)$$

Another requirement on the pinning potential is that the strength of the pinning force should take a random value within a specified range. This requirement can be satisfied by assuming that the coefficient U_p in equation (2.5c) takes a random value inside the range given by

$$(0 <) U_{\text{pm}} \leq U_p \leq U_{\text{pM}}, \quad (2.5d)$$

where U_{pm} and U_{pM} are the pinning parameters characterizing the strength of pinning forces of the assembly of N_p pinning centers.

Let us again notice that the three kinds of stochastic characteristics including in the present set of the Langevin equations are specified concretely by equations (2.2a), (2.2b), (2.4b), and (2.5d), where (2.4b) specifies the range of the random distribution for the position of the pinning centers, and (2.5d) specifies the range of the random distribution for the strength of the pinning forces.

2.3 Normalization of the Langevin equations

Since the general solution of the starting equations, proposed in the present paper as a set of one-dimensional Langevin equations, can hardly be obtained by theoretical analysis, the numerical investigation of these equations seems to be another useful method.

In the present study, therefore, we intend to make the numerical investigation for two kinds of purposes:

Purpose 1: The propriety of the present simplification to start from a set of one-dimensional Langevin equations instead of three-dimensional Langeven equations can be checked by comparing the numerically solved results with the observed results for relatively large values of the current density, J , for which the scaling law predicted by Fisher et al. [1] has been confirmed by many measurements, as mentioned in the section 2.1.

Purpose 2: As the final purpose of the present numerical investigation, we intend to obtain the asymptotic behavior of the induced electric field, E , in the limit of $J \rightarrow 0$. For this purpose, the numerical calculation for the ultra-long time relaxation of the induced electric field, E , is necessary.

In order to carry out the numerical investigations for the above-mentioned purposes, let us at first assume that the concerning glassy state of fluxoids is realized by applying the external magnetic field with the flux density, B_e , to the sample. Since the magnetic flux of a single fluxoid resulting from the rotational motion of the Cooper-pair electrons

with the electric charge of $2e$ is given by the flux quantum, $\Phi_0 = h_p/2e$ with the Planck constant, h_p , then the average interval between successive fluxoids, a , is given by

$$a = (\Phi_0/B_e)^{1/2}. \quad (2.6a)$$

Since the largest pinning force among the pinning potentials given by equation (2.5c) with (2.5d) is given by $4U_{PM}/\ell_L a$, any fluxoid can move to the direction of the driving force across the pinning potentials, when the driving force, $\Phi_0 J$ exceeds the critical value, $\Phi_0 J_{CM}$, given by

$$\Phi_0 J_{CM} = 4U_{PM}/\ell_L a. \quad (2.6b)$$

Since the effect of the pinning potentials becomes to be disregarded at $J \gg J_{CM}$, the flux-flow electric resistivity approaches ρ_F , which is independent of J , at $J \gg J_{CM}$. Thus ρ_F has been regarded as one of the measurable physical parameters characterizing the fluxoid system. Taking account of the fact that any fluxoid is moving with the same velocity, V_0 , at $J \gg J_{CM}$, the resistive force against to the driving force $\Phi_0 J$, is only the viscous-drag force given by $\eta \partial X_j / \partial t = \eta V_0$. With the aid of the relation given by $E = B_e V_0 = \rho_F J$, then we get

$$\eta = B_e \Phi_0 / \rho_F. \quad (2.6c)$$

Now let us define the dimension-less variables by

$$x_j = X_j/a, \quad x = X/a, \quad (2.7a)$$

$$\tau = t/t_0, \quad (2.7b)$$

$$\xi_j(\tau) = \Xi_j(t_0\tau)/\beta_T; \quad \beta_T \equiv (2\eta k_B T/t_0)^{1/2}/\Phi_0 J_{CM}, \quad (2.7c)$$

and also define the dimension-less parameters as

$$e_v \equiv \eta a / t_0 \Phi_0 J_{CM}, \quad (2.8a)$$

$$k_s \equiv Ka / \Phi_0 J_{CM} = C_{66} a / \Phi_0 J_{CM}, \quad (2.8b)$$

$$u_p \equiv U_P / U_{PM}, \quad (2.8c)$$

$$j_e \equiv J / J_{CM}, \quad (2.8d)$$

where C_{66} is the shearing elastic constant [2] of the fluxoid system.

If we divide the Langevin equation given by equation (2.1) by $\Phi_0 J_{CM}$, we get the normalized Langevin equation described by the dimension-less variables and dimension-less parameters as

$$e_v \frac{\partial x_j(\tau)}{\partial \tau} = k_s (2x_j(\tau) - x_{j+1}(\tau) - x_{j-1}(\tau)) + \sum_{p=1}^{p=N_p} \Delta(x_j(\tau); x_p) \frac{\partial u_{pj}(x_j(\tau); x_p \Delta x_p)}{\partial x_j(\tau)} + \beta_T \xi_j(\tau) + j_e, \quad (2.9a)$$

with the aids of the expressions given by

$$\Delta(x_j(\tau); x_p) = 1; \quad p \leq x_j(\tau) \leq p+1, \\ = 0; \quad \text{otherwise}, \quad (2.9b)$$

$$u_{pj}(x_j; x_p) = -u_p \left(\frac{x_j - p}{\Delta x_p} \right)^4 \left(\frac{1 - (x_j - p)}{1 - \Delta x_p} \right)^4 \\ \times \left[\frac{3\Delta x_p (1 - (\Delta x_p)^2)}{\Delta x_p (1 - (x_j - p))^2 + 2(1 - \Delta x_p)^2 (x_j - p)} \right]^6. \quad (2.9c)$$

In the above expressions, u_p and Δx_p take random values respectively inside the respective ranges, given by

$$u_{PM} (\equiv U_{PM}/U_{PM}) \leq u_p \leq 1, \quad (2.10a)$$

$$\frac{\ell_L}{2} \leq \left| \frac{1}{2} - \Delta x_p \right| \leq \frac{1 - \ell_L}{2}. \quad (2.10b)$$

It is to be noted that the numerical investigation for the Purpose 1 can be carried out safely by neglecting the thermal fluctuation force given by $\beta_T \xi_j(\tau)$, and hence the value of t_0 , that is a measure of the time interval appeared in equation (2.7b), can be chosen appropriately. For the numerical investigation for the Purpose 2, on the other hand, we should calculate the relaxation behavior of $x_j(\tau)$ over ultra-long time interval in the presence of the thermal fluctuating force. Then our hope to choose the value of t_0 as large as possible will be confined to the ratio of t_0 to the time scale characterizing the thermal fluctuation of fluxoids.

3. Formal expression for the electric resistivity at $J \rightarrow 0$

For the convenience of the following theoretical analysis of the set of Langevin equations, (2.9a), let us rewrite it in a simpler form as

$$e_v \frac{\partial x_j(\tau)}{\partial \tau} = \frac{\partial}{\partial x_j} u_{\text{eta}}(\{x_j\}) + \frac{\partial}{\partial x_j} u_{\text{pin}}(\{x_j\}) + \beta_T \xi_j(\tau) + j_e. \quad (3.1)$$

Let us introduce a probability distribution function, $P(\{x_j(\tau)\}, \tau)$. Then the Fokker Planck equation corresponding to equation (3.1) is given by

$$\frac{\partial}{\partial \tau} P = LP; \quad (3.2a)$$

$$L = L_0 + L_1; \quad (3.2b)$$

$$L_0 \equiv \frac{1}{e_v} \sum_j \frac{\partial}{\partial x_j} \left[\beta_T \frac{\partial}{\partial x_j} + \frac{\partial u(\{x_j\})}{\partial x_j} \right]; \quad (3.2c)$$

$$u(\{x_j\}) \equiv u_{\text{eta}}(\{x_j\}) + u_{\text{pin}}(\{x_j\}), \quad (3.2d)$$

$$L_1 \equiv -\frac{j_e}{e_v} \sum_j \frac{\partial}{\partial x_j}. \quad (3.2e)$$

For the purpose to investigate the behavior of the electric resistivity in the limit of $J \rightarrow 0$, we have only to notice the linear response of P with respect to J . Since L_1 is only the operator that depends explicitly on J , as can be seen by equation (3.2e), let us at first notice the operator identity given by

$$\exp(\tau L) = \exp(\tau L_0) + \int_0^\tau d\tau' \exp[(\tau - \tau') L_0] L_1 \exp(\tau' L), \quad (3.3a)$$

where the electric current with the current density, $J \equiv J_{CM} j_e$, is assumed to be applied at $\tau = 0$.

Since the initial distribution of fluxoids at $\tau = 0$ should be chosen as the equilibrium distribution in the absence of the applied electric current with the current density, J , then the integral form of the probability distribution function, $P(\{x_j(\tau)\}, \tau)$, that is abbreviated by $P(\tau)$ for simplicity, is

given by

$$P(\tau) = P_{\text{eq}} + \int_0^\tau d\tau' \exp(\tau' L_0) L_1 P(\tau - \tau'), \quad (3.3b)$$

where P_{eq} represents the probability distribution function in the absence of the external current. Thus the stationary state under the presence of J is given by

$$P(\infty) = P_{\text{eq}} + \int_0^\infty d\tau \exp(\tau L_0) L_1 P(\infty). \quad (3.3c)$$

If the probability distribution in the stationary state up to the linear response with respect to the external current density, J , is denoted by $P_1(\infty)$, the expression for $P_1(\infty)$ can be obtained by approximating $P(\infty)$ on the right-hand side of equation (3.3c) by P_{eq} . Then we get

$$P_1(\infty) = P_{\text{eq}} + \int_0^\infty d\tau \exp(\tau L_0) L_1 P_{\text{eq}}. \quad (3.3d)$$

Now let us define the averaged position of the fluxoids by

$$x_{\text{av}}(\tau) = \frac{1}{N_f} \sum_{j=1}^{j=N_f} x_j(\tau). \quad (3.4a)$$

Since the electric field, E , is defined by

$$\begin{aligned} E(\tau) &= B_e \frac{a}{l_0} \frac{d}{d\tau} \langle x_{\text{av}}(\tau) \rangle \\ &= \rho_F J_{\text{cm}} e_V \frac{d}{d\tau} \langle x_{\text{av}}(\tau) \rangle, \end{aligned} \quad (3.4b)$$

then the electric resistivity, ρ , is defined by

$$\rho = \frac{E}{J} = \rho_F \frac{e_V}{j_e} \frac{d}{d\tau} \langle x_{\text{av}}(\tau) \rangle. \quad (3.4c)$$

With the aid of the Fokker Planck equation (3.2a), we get the formal relation:

$$\begin{aligned} \frac{d}{d\tau} \langle x_{\text{av}}(\tau) \rangle &= \Pi_j \int_0^\infty dx_j [x_{\text{av}} L \exp(\tau L) P] \\ &= \Pi_j \int_0^\infty dx_j [L^\dagger x_{\text{av}} \exp(\tau L) P]. \end{aligned} \quad (3.4d)$$

For the present purpose, we can safely approximate P in equation (3.4d) by $P_1(\infty)$ given by equation (3.3d).

As for the contribution of P_{eq} , which is the first term of $P_1(\infty)$, to the normalized velocity, $d\langle x_{\text{av}} \rangle/d\tau$, it can be estimated from the following consideration, while its expression given by (3.4d) looks to be complicated: Since the distribution of fluxoids is kept to be in the equilibrium distribution at $\tau=0$, the resistive force against to the driving force, j_e is only the viscous drag force, $e_V \nu$. Then any fluxoid as well as the averaged position of fluxoids moves with a constant velocity given by $\nu = j_e/e_V$. From equations (3.4c), therefore, the contribution of P_{eq} in (3.4d) to the electric resistivity, ρ , is simply given by ρ_F .

Then the formal expression for the electric resistivity in the stationary state in the limit of $J \rightarrow 0$ is given by

$$\begin{aligned} \rho &= \rho_F \\ &+ \rho_F \frac{e_V}{j_e} \int_0^\infty d\tau \Pi_j \int_0^\infty dx_j [L^\dagger x_{\text{av}} \exp(\tau L_0) L_1 P_{\text{eq}}]. \end{aligned} \quad (3.5a)$$

Since the thermal fluctuation force is balanced with the force caused by the total potential, u , defined by equation

(3.2d), for the equilibrium state, P_{eq} , we can put from (3.3c) as

$$\beta \tau \sum_j \frac{\partial}{\partial x_j} P_{\text{eq}} + \sum_j \frac{\partial u(\{x_j\})}{\partial x_j} P_{\text{eq}} = 0. \quad (3.5b)$$

In addition, the following relation for any potential, $u(\{x_j\})$, can be derived easily:

$$\sum_j \frac{\partial}{\partial x_j} u(\{x_j\}) = \frac{\partial}{\partial x_{\text{av}}} u(\{x_j\}). \quad (3.5c)$$

With the aids of equations (3.5b), (3.5c), and equation (3.2d), the expression for the resistivity, ρ , in the stationary state in the limit of $J \rightarrow 0$ is reduced to

$$\begin{aligned} \rho &= \rho_F \\ &- \frac{\rho_F}{\beta \tau} \int_0^\infty d\tau \Pi_j \int_0^\infty dx_j [L_0^\dagger x_{\text{av}}] \exp(\tau L_0) \left[-\frac{\partial u(\{x_j\})}{\partial x_{\text{av}}} \right] P_{\text{eq}}. \end{aligned} \quad (3.5d)$$

It is to be emphasized that the above expression represents the flux-creep resistivity in the limit of $J \rightarrow 0$, because any operator depending on J explicitly is not contained.

Then the general theoretical prediction proposed by Fisher et al. [1] may be concluded to be valid only when the right-hand side of the above expression is proved to be zero, even for the example of pinning potential that appears in equations (2.9a), (2.9b) and (2.9c).

4 . Summary

In this paper, we proposed the starting equations for the theoretical investigations on the problem to answer the question: Does the electric resistivity, ρ , approach zero as the applied electric current approaches zero, in the glassy state of the quantized fluxoids in a type 2 superconductor ?

The proposed starting equations are given by a set of the Langevin equations, each of which describes the motion of each fluxoid. Since these starting Langevin equations contain three kinds of forces, each of which has a stochastic nature, we further proposed the starting Fokker Planck equation.

With the aid of this Fokker Planck equation, we derived the formal expression for the electric resistivity, ρ , in the limit of the current density, J , of the applied electric current approaches zero.

Since the obtained expression is only the formal expression, we need further theoretical derivation for the example of the pinning potential given in the sections 2.1 and 2.2. The result will be reported in the following papers.

ACKNOWLEDGEMENT

We would like to acknowledge the fact that the present

work is supported by RESEACH GRANT No. 20548381 of the Japan Society for Promotion of Science.

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