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Multi-Selection Information Diffusion Game
with Revealed Users' Preferences

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Abstract

In this research, the authors propose an extension to information diffusion game with revealed users' preferences. An information diffusion game consists of a set of external players (interested parties), and a network of users (vertices). Our approach differs from the literature in two aspects: first, unlike the cancelled-out assumption in Alon et al. (1), each vertex has its own preference regarding the information released by the players, and will take the most preferred one. Second, instead of selecting only one vertex as the seed at the initial stage, the players are allowed for a more general strategy, i.e., multiple selections. Two experimental studies on the existence of pure Nash equilibria with this novel algorithm are conducted by an application called IDG (Information Diffusion Game) simulator, which is developed in Python.

Keywords: Information diffusion game, revealed preferences, multiple selections, pure Nash equilibrium, maximin strategy, IDG simulator

1. INTRODUCTION

Information diffusion game is constituted of a network, and a set of external players. Each player chooses an initial vertex as the seed to propagate her information and expects it to spread throughout the whole network. This is very common in our society. One example is rally in presidential election. Candidates have to carefully select the initial state(s) to disseminate their political proposals in order to acquire the most votes.

Information diffusion game differs from Voronoi games (Durr and Thang, 2007 (2)) in the vertex assignment rules. Instead of assigning each vertex to the closest facility, information will spread along the edges in the network and a player's final payoff is defined by the number of vertices taken by her information. This game was first introduced by Alon et al. (1) with focus on the relation between the existence of pure Nash equilibrium and the diameter of the network. Particularly, they gave a proof that pure Nash equilibrium exists if the diameter is at most two. Later, Takehara et al. (5) corrected the statement of this theorem by a counterexample. In subsequent researches, sufficient conditions for the existence of pure Nash equilibrium in path graph (6), in a tree structure (4), in weighted cycles (7), with weighted vertex (3) were also explored.

In this paper, we propose a novel and more natural algorithm incorporating the vertices' preferences, and each vertex will be "colored" by its most preferred information. In the case that a vertex has exactly the same preferences regarding two or more information, it is assumed to take one randomly with uniform distribution. Another extension from the literature lies in the strategy, where players are allowed to make multiple selections in the initial stage.

The remainder of this paper is organized as follows. In Section 2, we describe the algorithm for the multi-selection information diffusion game with revealed preferences. In Section 3, the design of IDG simulator is documented function by function with flowcharts. Section 4 gives the simulation results for both single-selection and 2-selection information diffusion games, and explores the existence of pure Nash equilibrium. Section 5 concludes this paper.

2. MULTI-SELECTION INFORMATION DIFFUSION GAME WITH REVEALED USERS' PREFERENCES

2.1. THE MODEL

Let $G = <V, E>$ represent the undirected network, where V is the set of vertices, and E is the set of edges. Let $N = \{1, \ldots, n\}$ be the set of external players (interested parties), and

$C = \{c_1, \ldots, c_n, w\}$ be the set of information colors, where c_j corresponds to player j's information, and w denotes the null information state in white. Let $P = (P_1, \ldots, P_v)^T$ represent the preference distribution vectors of all vertices, where $P_i = (p_{i1}, \ldots, p_{in})^T$ is vertex i's preferences regarding the information released by all players, and $\sum_{j=1}^{n} p_{ij} = 1$ is satisfied.

Moreover, let $S_j \subseteq V$ denote the strategy of player j at Time 1. Let $N_i \subseteq N$ denote the set of players selecting vertex i at Time 1, and $N_i^* = \{m\}_{m \in N_i}$ denote the set of players such that $p_{im} = \max\{p_{ik}\}_{k \in N_i}$. Also, let $C_i^t \subseteq C$ denote the color set (excluding white) of vertex i's neighbors at the end of Time t, and let $\tilde{N}_i^{t*} = \{m\}_{c_m \in C_i^t} \subseteq N$ denote the set of players such that $p_{im} = \max\{p_{ik}\}_{c_k \in C_i^t}$. Finally, let $V_j^t \subseteq V$ denote the set of vertices in c_j at the end of Time t. The multi-selection information diffusion game with revealed preferences unfolds as below:

- **Time 0**: all vertices are initialized with white color.
- **Time 1**: each player $j \in N$ selects one or multiple vertices as her strategy S_j, then
  - If vertex i is selected by null player ($|N_i| = 0$), it will remain in white.
  - If vertex i is selected by only one player $j(|N_i| = 1)$, it will be colored in c_j.
  - If vertex i is selected by two or more players ($|N_i| \geq 2$), it will be colored in c_j with probability $1/|N_i^*|$, where $j \in N_i^*$.
  - The algorithm ends if $V \setminus \{\cup V_j^1\}_{j \in N} = \emptyset$.
  - The payoff for player $j \in N$ at Time 1 is $u_j^1 = |V_j^1|$.
- **Time t** ($t \geq 2$): vertex $i \in V \setminus \{\cup V_j^{t-1}\}_{j \in N}$ will be colored following the rules below:
  - If vertex i's neighbor vertices are all in white ($C_i^{t-1} = \emptyset$), it will remain in white.
  - If vertex i's neighbor vertices are all in $c_j \in C_i^{t-1}$ ($|C_i^{t-1}| = 1$), it will be colored in c_j.
  - If $|C_i^{t-1}| \geq 2$, vertex i will be colored in c_j with probability $1/|\tilde{N}_i^{(t-1)*}|$, where $j \in \tilde{N}_i^{(t-1)*}$.
  - The algorithm ends if $V \setminus \{\cup V_j^t\}_{j \in N} = \emptyset$.
  - The payoff for player $j \in N$ at Time t is $u_j^t = |V_j^t|$.

2.2. PURE NASH EQUILIBRIUM

For multi-selection information diffusion game with revealed preferences, the strategy set for any player $j \in N$ could be denoted by V. Stated simply, players only have to select one or more target

vertices at Time 1, then their final payoffs can be deduced following the information diffusion algorithm. The pure Nash equilibrium is defined as below:

Definition 1 (Pure Nash Equilibrium): A strategy profile $S^* = (S_1^*, \ldots, S_n^*) \in V^n$ is a pure Nash equilibrium if no unilateral deviation in strategy by any single player is profitable for that player, that is:

$$u_j(S_j^*, S_{-j}^*) \geq u_j(S_j, S_{-j}^*) \ \forall j \in N, \forall S_j \in V \qquad (1)$$

2.3. MAXIMIN STRATEGY

Besides the pure Nash equilibrium as in the literature, the worst-case payoff is also concerned in this research. Player j's maximin strategy is a strategy that maximizes j's worst-case payoff, in the situation where all other players happen to play the strategy which causes the greatest harm to player j. The maximin value (or safety value) of the game for player j is that minimum payoff guaranteed by a maximin strategy. It can be formally defined as below:

Definition 2 (Maximin Strategy): The maximin strategy for player $j \in N$ is $\arg\max_{S_j} \min_{S_{-j}} u_j(S_j, S_{-j})$, and the maximin value for player j is $\max_{S_j} \min_{S_{-j}} u_j(S_j, S_{-j})$.

3. DESIGN OF IDG SIMULATOR

An application, called IDG simulator hereafter, has been developed in Python. At the current stage, IDG simulator has implemented the multi-selection information diffusion algorithm with revealed users' preferences in 2-player case, which is extensible to multi-player cases. Theoretically, it is capable of handling networks with uncountable vertices and edges, and outputting the pure Nash equilibria and maximin strategies for all players. In this section, the design of IDG simulator is documented function by function with flowcharts.

3.1. NETWORK GENERATION

The network is generated by the number of vertices as well as the connection information represented by checkbox. As is shown in Figure 1, n is the number of players, i and j are loop variables for vertices, and $con[]$ is an array that stores whether a pair of vertices is connected or not. The network generation flow could be briefly described as below:

- Initialize all the variables.
- Retrieve the number of players entered in the text box, and plug it into variable n.
- Checkboxes are placed on the interface using nesting of i and j.
- Connection information is stored in con.

3.2. MULTI-SELECTION INFORMATION DIFFUSION GAME WITH REVEALED USERS' PREFERENCES

The program for multi-selection information diffusion game with revealed users' preferences follows the algorithm introduced in Section 2.1. The implementation flow shown in Figure 2 could be briefly summarized as below:

- Initialize the graph.
- By importing the CSV, connection information and vertices' preferences are plugged into array variables, respectively. The vertex preference is generated following normal distribution based on the registered statistics such as mean and variance.
- Each player selects one or more vertices as her strategy, and the selected vertices are colored following the rules in **Time 1**.
- At consecutive stages, each colored vertex sets a Flg to all its adjacent white vertices, and the white vertices are colored following the rules in **Time t** ($t \geq 2$).
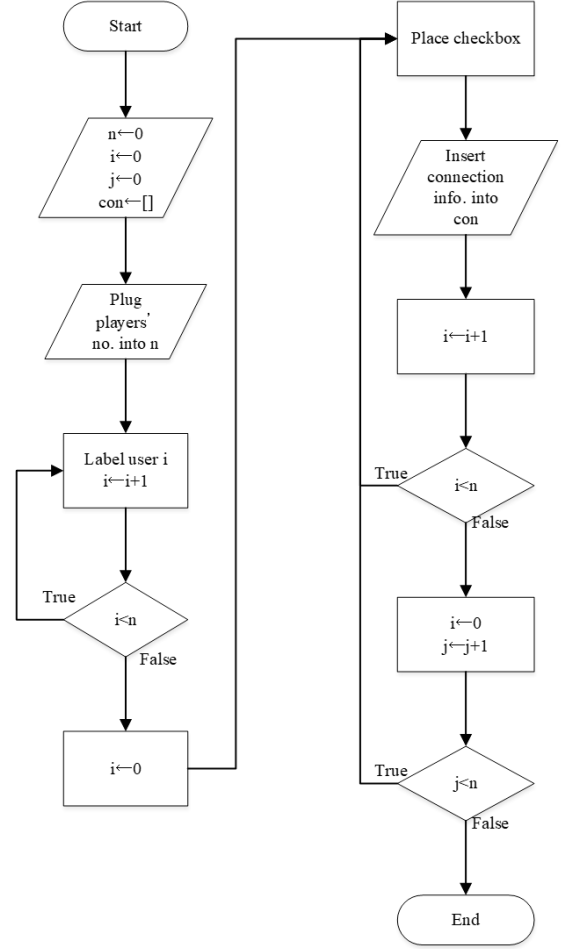

Fig. 1. flowchart: network generation

- The diffusion process ends if the counted number of colored vertices does not increase anymore, and the number of vertices in each player's information color is her final payoff.

Moreover, each vertex is assignable with a customized weight, which will be reflected in the final payoff (3). Stately intuitively, if a vertex with weight 20 is colored in c_j, the final payoff of player j will be increased by 19 compared to the network with identical weight 1 for all vertices.

3.3. MULTI-SELECTION INFORMATION DIFFUSION GAME WITHOUT USERS' PREFERENCES

For comparison study, we also incorporate the multi-selection information diffusion game without users' preferences (Figure 3) into IDG simulator. The only difference lies in the coloring rule of a white vertex when two or more information confronting each other. As is proposed in Alon et al. (1), these information will be cancelled out and the white vertex will be colored in gray. No more information could diffuse through the gray vertex to its neighbor white vertices.

4. EXPERIMENTAL STUDY

In this section, two experimental studies are conducted with IDG simulator to verify the existence of pure Nash equilibria and calculate each player's maximin strategy.
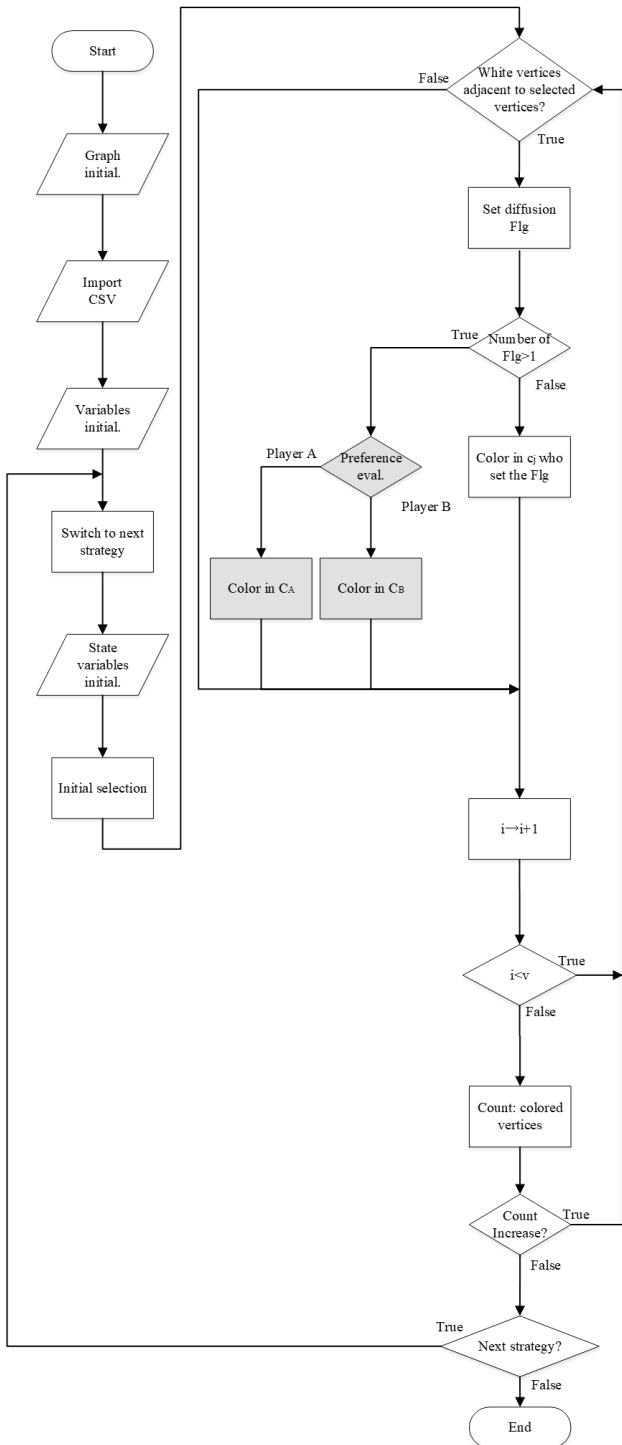
Fig. 2. flowchart: multi-selection information diffusion game with revealed users' preferences

Fig. 3. flowchart: multi-selection information diffusion game without revealed users' preferences

4.1. STRATEGY ANALYSIS OF 2016 U.S. PRESIDENTIAL ELECTION

In this experiment, IDG simulator is applied in the rally strategy analysis of 2016 U.S. presidential election, which is assumed to be a single-selection information diffusion game. One focus of this experimental 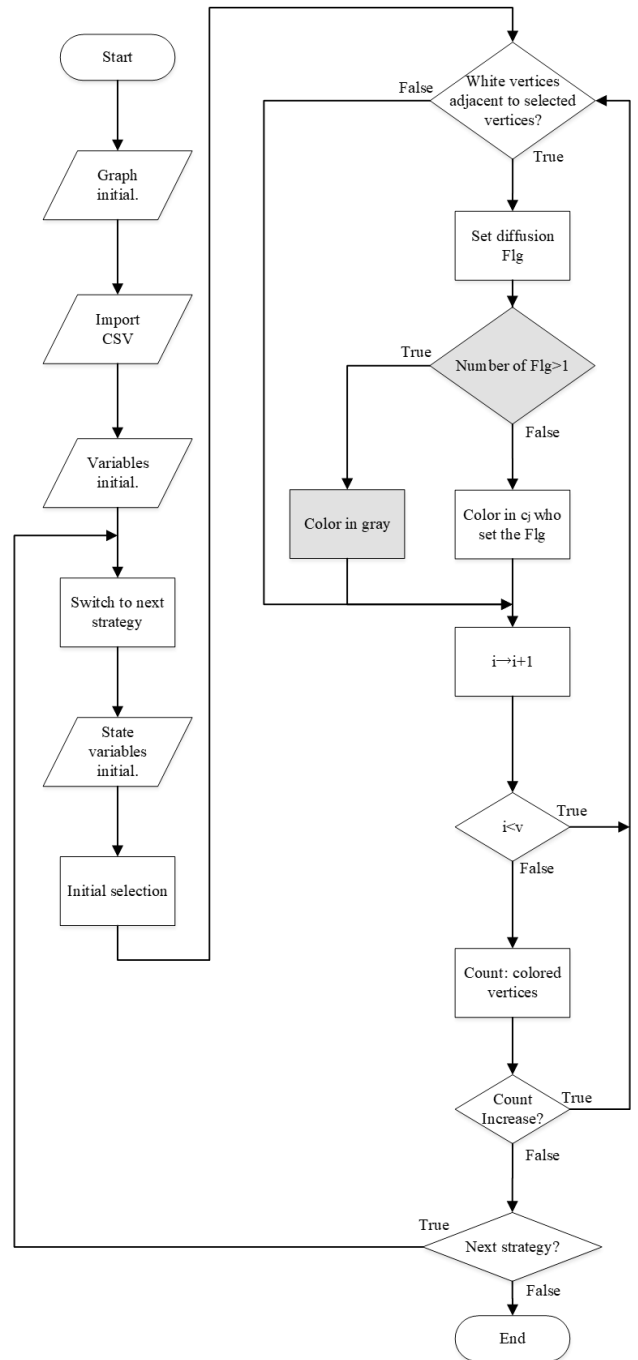study is the impact of local media in the information diffusion process. In other words, suppose Clinton and Trump target only one state before the official campaign and expect their political proposals might be spread throughout the whole country via local media reachable to its neighbor state. Which state should be the starting point? Hence the network is abstracted from the geographical information of U.S., which is constituted of 51 vertices. The preference distribution information is aggregated from the statistics of historical popular voting results (1992-2012), and the weight is in accordance with the electoral number in each state.

Table 1. experimental results for 2016 U.S. presidential election

	Case 1	Case 2	Case 3
Pure Nash equilibria	N/A	N/A	N/A
Avg. maximin values	(0, 0)	(23, 20)	(256, 205)
No. of maximin strategies / iteration	51	1-3	1-2

Table 2. distribution of maximin strategies

State	Case 2: Rep	Case 2: Dem	Case 3: Rep	Case 3: Dem
AR	2	0	0	7
CO	3	1	0	0
IL	0	0	0	2
KY	64	14	14	12
MD	0	21	0	7
MO	101	99	72	86
NE	6	0	0	0
OH	0	43	0	30
OK	31	0	10	0
PA	0	11	0	5
TN	16	1	101	29
VA	6	23	4	34
WI	0	1	0	0
WV	0	9	0	2

The experimental results of 200-iterated single-selection information diffusion game **(Case 1)** without revealed preferences, **(Case 2)** with revealed preferences and identical weights, **(Case 3)** with revealed preferences and distinct weights are summarized in Table 1. We notice that pure Nash equilibria do not exist in all cases due to the complexity of network and generated preference. In **Case 1**, the number of maximin strategies for each player in each iteration is 51 as there is always a cancelled-out payoff $(0, 0)$ if two players select the same state. However, maximin strategies in **Cases 2 and 3** are more focused on several specific states, i.e., Missouri, Kentucky, etc. (Table 2), and more insightful for players than that of case 1.

With visualized distribution of the maximin strategies in Figures 4-7, we can read that in order to guarantee the safety votes in the election, Republican and Democratic parties tend to adopt the following strategies:

- **Case 2:** both Republican and Democratic parties will select Missouri with top priority. However, to avoid the all-swipe situation by the opponent due to the generated preferences for certain states, i.e., the swing states, they may also deviate to the second-prior state. In other words, Republican and Democratic may deviate to Kentucky and Ohio, respectively.
- **Case 3:** Republican tend to select Tennessee, while Democratic tend to select Missouri at the initial stage.

We also notice that the high-frequent states in the maximin strategy distribution either have the most connections to other states, or are connected to the most connected states. They also have a feature that the preference mean is very close to 50%.

4.2. 2-SELECTION INFORMATION DIFFUSION GAME WITH RE-VEALED USERS' PREFERENCES

In this section, an experimental study for 2-selection case is conducted, where the number of vertices is assumed to be 9. As an abstract case, the preference for each vertex is generated with uniform distribution between 0 and 1. In a general multi-selection case, it could also be generated with multinomial distribution. The payoff with pure Nash equilibria in 200 iterations is summarized in Table 3, in which player 1 tends to select vertices 1 and 8, while player 2 tends to select vertices 2 and 3 at the initial stage. Moreover,
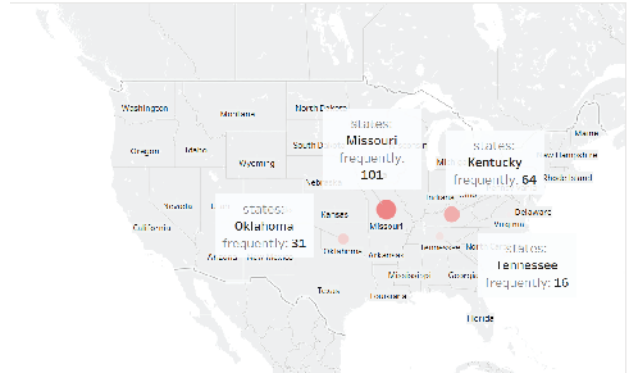

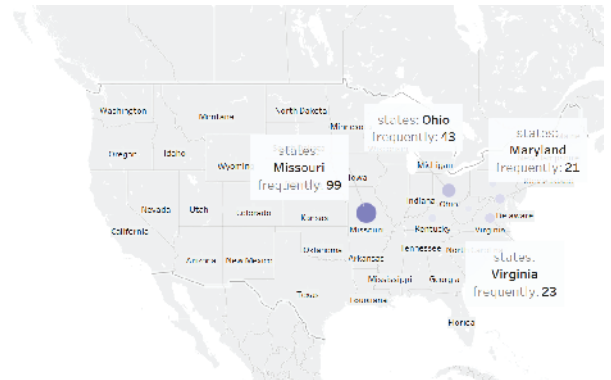Fig. 4. **Case 2**. distribution of maximin strategies for Republican

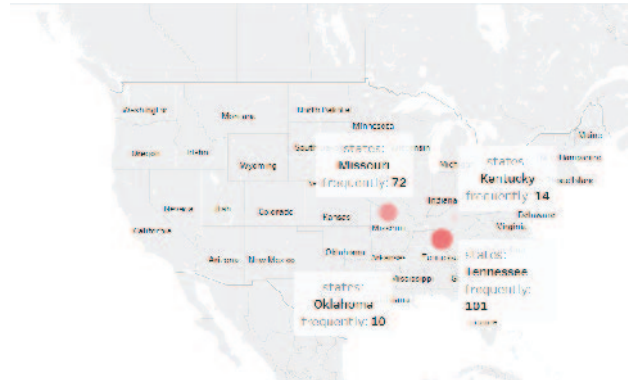Fig. 5. **Case 2**. distribution of maximin strategies for Democratic

Fig. 6. **Case 3**. distribution of maximin strategies for Republican

the distribution of maximin strategies for both players are shown in Figure 8, and we notice that $(4, 8)$ is the most selected strategy for both players to maximize their worst-case payoff.

5. SUMMARY

In this paper, we propose a multi-selection information diffusion game with revealed users' preferences. The novelty in our model lies in three aspects: (1) endogenous diffusion process; (2) fairness of users' involvement; (3) multi-selection at initial stage. All these features reflect the characteristics of a natural social interaction. Referring to the experimental results, the authors are currently working on the theoretical proof for the existence condition of pure Nash equilibrium in this game.
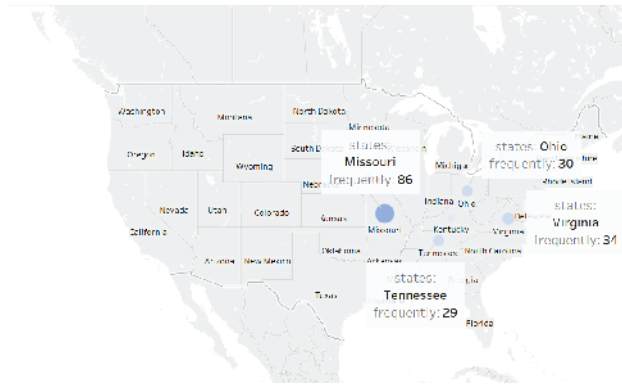
Fig. 7. **Case 3**. distribution of maximin strategies for Democratic

Table 3. pure Nash equilibria in 2-selection information diffusion game with revealed users' preferences

Iteration	Strategy of player 1	Strategy of player 2	Payoff
1	(2, 4)	(3, 8)	(5, 4)
42	(1, 8)	(2, 3)	(5, 4)
42	(1, 9)	(2, 3)	(5, 4)
126	(1, 8)	(2, 4)	(6, 3)
149	(1, 8)	(2, 4)	(6, 3)
190	(6, 8)	(2, 4)	(4, 5)

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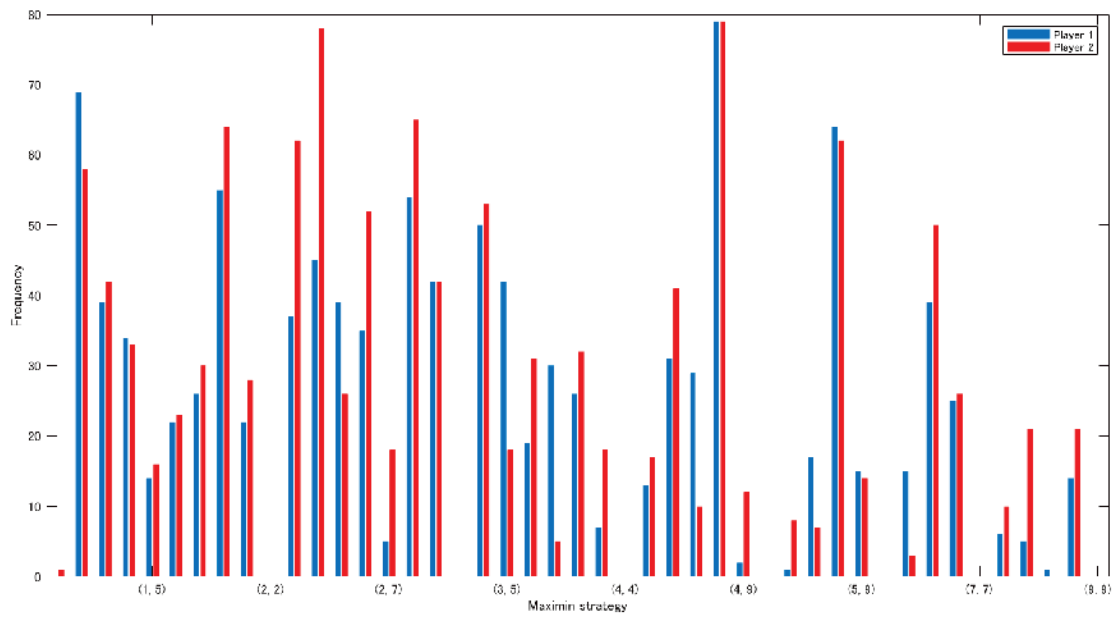
Fig. 8. distribution of maximin strategies in 2-selection information diffusion game with revealed users' preferences