Effect of the Aspect Ratio of a Superconducting Tape on the AC Loss in Perpendicular External Magnetic Field

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Abstract

Recently, intensive efforts to decrease the width of the high-T_c cuprate superconducting tape for the coated conductors have been made for decreasing the AC loss. It has been found experimentally that the AC loss of the former type of superconducting tape, with the width extremely larger than the thickness, can be described well by the theoretical expression for the superconducting thin strip with the infinite width proposed by Brandt and Indenbom. However, the AC loss in the recent types of tapes, with a much narrower width, is expected to show a noticeable deviation from their expression. In this paper, based on the critical state model, a theoretical expression for the dependence of the AC loss on the aspect ratio in the cross section of the superconducting tape is proposed. Effect of the magnetic-flux-density dependence of the critical current density on the AC loss is also discussed.

Keywords: superconducting tape, alternating perpendicular field, AC loss, aspect ratio, thin strip, critical state model, critical current density

1. Introduction

The development of the superconducting coated conductors for the electro-magnetic power devices has been progressing actively in Japan and other countries. While the present situation may be fairly satisfactorily so far as the request on the high current density from the application side is concerned, the problem of the optimization of the AC loss still remains as one of the important tasks to be developed.

The superconducting tape for the coated conductors composed of the YBCO-series high-T_c cuprate superconductors developed previously has the cross section with, typically, the thickness of a few μm and the width of several mm, for the purpose of obtaining a high current density. As the first step for decreasing the AC loss, therefore, these previous type of superconducting tapes have been tried to cut into many strips with much narrower width in retaining the thickness to be a few μm.

The superconducting tapes under developing for the coated conductors are usually the c-axis oriented ones having a strong anisotropy for the critical current density. Amemiya et al. [1] showed experimentally that the observed data of the AC loss in the previous type of tapes under the applied oblique magnetic field can be scaled to a single master curve when the observed values of the AC loss are re-plotted against the perpendicular component, $H_e \sin\theta$, of the applied oblique magnetic field, H_e, where θ is the angle from the broadest surface of the thin superconducting tape. Whereas this kind of scalability can be attributed to the strong anisotropy in the critical current density, they

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also confirmed [1] that the obtained master curve showed a good agreement with the theoretical expression proposed by Brandt and Indenbom [2] for the AC loss of the thin superconducting tape with an infinite width in the applied perpendicular magnetic field. An essentially the same conclusion was also obtained by Tsukamoto [3] based on a more detailed discussion.

If the aspect ratio of the width, $2D_X$, to the thickness, $2D_Y$, in the cross section of the superconducting tape is defined by $1/\alpha \equiv D_X/D_Y$, the value of $\alpha \equiv D_Y/D_X$ for the above-mentioned previous type of superconducting tape is much less than 10^{-3}. Then the theoretical expression for the AC loss derived by Brandt and Indenbom [2] for the limit of $\alpha \sim 0$ is safely applicable to the case of previous type of superconducting tape. So far as the superconducting tape is intended to use in the coated conductors, however, the optimum value of α should be chosen as $10^{-3} \ll \alpha \ll 1$ for the purposes of decreasing the AC loss of the tape in keeping the high critical current density. Then the expression for the AC loss derived for the limit of $\alpha \sim 0$ [2] is expected to show a noticeable deviation from the observed data for the optimized superconducting tapes.

In this paper, a theoretical expression for the AC loss in the superconducting tape under the applied perpendicular magnetic field is derived for the case of $\alpha \ll 1$, based on the critical state model [3], which was also adopted in the derivation of the theoretical expression by Brandt and Indenbom [2]. Since the critical current density at a given spatial position generally depends on the magnetic flux density of the concerning position [4], the effect of the magnetic flux density dependence of the critical current density on the AC loss is also discussed theoretically, while Brandt and Indenbom [2] discussed only the limiting case that the critical current density is constant independently of the magnetic flux density.

2. Theoretical expression for the AC loss in the superconducting tape with constant critical current density

Let us consider the superconducting tape having infinite length along the Z axis, the width of $2D_X$ along the X axis, and the thickness of $2D_Y$ along the Y axis. When the aspect ratio in the cross section of the tape is defined by $D_X/D_Y \equiv 1/\alpha$, then α is given by

$$\alpha \equiv D_Y/D_X. \tag{2.1}$$

The purpose of the present theoretical investigation is to derive a theoretical expression for the AC loss in the superconducting tape having the cross section of $\alpha \ll 1$ under the perpendicular magnetic field applied to the Y-direction.

For this purpose, we hereafter use the normalized coordinates defined by

$$x \equiv X/D_X, \quad y \equiv Y/D_Y = Y/\alpha D_X. \tag{2.2}$$

Furthermore, we adopt the so-called critical state model [4, 5] for the transport electric current induced by the applied magnetic field, because this model has been known to describe well the electro-magnetic properties in both the low-T_C metallic and the high-T_C cuprate superconducting materials [6].

According to the critical state model, the density of the induced electric current, J, is related to the critical current density, J_C, as [5]

$$J(x, y) = \lambda J_C(B(x, y)), \tag{2.3a}$$

where J_C is generally given by a function of the flux density, $B = B(x, y)$. In equation (2.3a), λ takes the value of $+1$ or -1 according as whether the direction of the induced current is the positive or the negative direction of the Z axis, and takes the value of 0 in the region where the current is not induced.

In this section, let us confine the discussion to a special case that the critical current density has a constant magnitude independent of the magnetic flux density. In this special case, equation (2.3a) is reduced to

$$J(x, y) = \lambda J_C. \tag{2.3b}$$

This type of the simplified model has also been adopted by Brandt and Indenbom [2] in the derivation of the theoretical expression for the AC loss in the thin superconducting tape with $\alpha \sim 0$.

The effect of the B-dependence of the critical current density on the AC loss will be discussed in the section 3.

2. 1 The shape of the flux front in the initial magnetization process

When the magnetic field, H_e, is initially applied to the superconducting tape in the Y-direction and is increased from zero, the magnetic flux begin to penetrate into the tape. Let us denote the shape of the flux front in this initial magnetization process by

$$x = x_F(y), \tag{2.4}$$

and define the complex flux density, $\tilde{B}(\tilde{z})$, at the complex position, $\tilde{z} \equiv x + i\alpha y$, by

$$\tilde{B}(\tilde{z}) \equiv B_Y(\tilde{z}) + iB_X(\tilde{z}) = \mu_0 H_e + \tilde{B}_J(\tilde{z})\ ;\ \tilde{z} \equiv x + i\alpha y, \tag{2.5a}$$

where the magnetic flux density induced by the transport electric current, $\tilde{B}_J(\tilde{z})$, is given from the Biot-Savart law by

$$\tilde{B}_J(\tilde{z}) = -\frac{\mu_0}{2\pi}\sum_\lambda \iint_{S_\lambda} D_Y \mathrm{d}y_0 \mathrm{d}x_0 \frac{\lambda J_C(x_0, y_0)}{x_0 + i\alpha y_0 - \tilde{z}}. \tag{2.5b}$$

In equations (2.5a) and (2.5b), μ_0 is the magnetic susceptibility of the vacuum, and S_λ is the region inside which the induced electric current has the current density of λJ_C, as shown schematically in figure 1a.

Then the shape of the flux front given by equation (2.4) can be determined from the condition that $\tilde{B}(\tilde{z})$ defined by equations (2.5a) and (2.5b) is zero everywhere in the region inside the flux front, S_0.

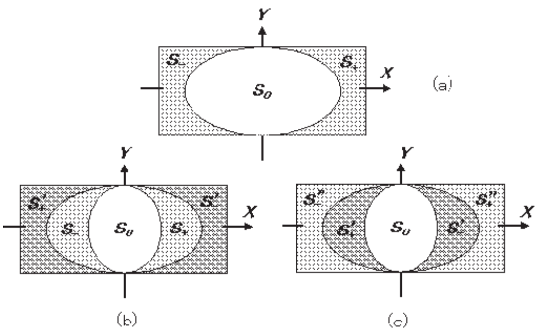

Fig. 1. Current regions S_+, S_-, S_0 having current density of λJ_C, where λ takes $+1, -1, 0$, respectively, (a) in the intial magnetization process, (b) in the following field decreasing process, and also (c) in the following field increasing process.

If we denote the shape of the flux front in the film limit of $\alpha \to 0$ by $x_{F0}(y) \equiv x_F(y\ ;\ \alpha \to 0)$, then $x_{F0}(y)$ is given by [2, 7]

$$|x_{F0}(y)| = |x_{F0}(0)| \frac{\tan \xi(y)}{\sqrt{A^2 + [\tan \xi(y)]^2}}\ ;\ \xi(y) \equiv \frac{\pi}{2}(1 - |y|), \tag{2.6a}$$

$$A^2 \equiv 1 - |x_{F0}(0)|^2. \tag{2.6b}$$

It has been shown [2, 7] that equation (2.6a) with equation (2.6b) is almost the exact solution of the critical state model for the film limit of $\alpha \to 0$. In fact, if we derive the expression for $\tilde{B}_J(\tilde{z})$ defined by equation (2.5b) using equation (2.6a) as described in Appendix , we get

$$\tilde{B}_J(x\ ;\ \alpha \to 0) = -\mu_0 H_{pB} \ln\left(\frac{1+A}{|x_{F0}(0)|}\right);$$
$$0 \le |x| < |x_{F0}(0)|, \tag{2.7a}$$

$$H_{pB} \equiv \frac{2}{\pi} J_C D_X \alpha = \frac{2}{\pi} J_C D_Y. \tag{2.7b}$$

Inserting equation (2.7a) into equation (2.5a), the shielding condition of the magnetic flux inside the flux front leads to

$$|x_{F0}(0)| \equiv \sqrt{1 - A^2} = \mathrm{sech}\ h_e, \tag{2.7c}$$

$$A \equiv \sqrt{1 - |x_{F0}(0)|^2} = \tanh h_e, \tag{2.7d}$$

where the normalized applied magnetic field, h_e, is defined by

$$h_e \equiv \frac{H_e}{H_{pB}}. \tag{2.7e}$$

If we assume that $|x_{F0}(0)|$ varies as the function of h_e as given by equation (2.7c), therefore, equation (2.7a) suggests that the magnetic flux is completely shielded in the region inside the flux front for the case of $\alpha \to 0$.

For the case of $\alpha \neq 0$, on the other hand, the exact solution of the critical state model has not yet been obtained. However, a good expression for the AC loss that explains well the observed results can be derived even by starting from the appropriately chosen approximate shape of the flux front [8], because the expression for the AC loss is derived after the double integrals of the shape of the flux front, as can be seen in the later sections.

Then let us consider the following expression as an approximate candidate of the shape of the flux front for the case of $\alpha \ll 1$:

$$|x_F(y)| = |x_{F0}(0)| \frac{\tan \xi(y)}{\sqrt{A^2[\Delta(\alpha,\ A)]^2 + [\tan \xi(y)]^2}}, \quad (2.8)$$

and determine the functional form of $\Delta = \Delta(\alpha,\ A)$ from the shielding condition that $\tilde{B}(\tilde{z})$ defined by equation (2.5a) becomes zero only at the center of the tape, $\tilde{z}=0$.

The detailed derivation of $\tilde{B}_J(0\ ;\ \alpha)$ defined by equation (2.5b) by using $x_F(y)$ given by equation (2.8) is shown in Appendix, and the resulting expression is given by

$$\tilde{B}_J(0\ ;\ \alpha) = -\mu_0 H_{pB}\left\{\ln\left(\frac{[1+A\Delta]\sqrt{1+\alpha^2}}{\sqrt{|x_{F0}(0)|^2+\alpha^2+\alpha A\Delta}}\right) + F(\alpha,\ |x_{F0}(0)|)\right\}; \quad (2.9a)$$

$$F(\alpha,\ |x_{F0}(0)|) = \frac{1}{\alpha}\tan^{-1}(\alpha) - \frac{|x_{F0}(0)|}{\alpha}\tan^{-1}\left(\frac{\alpha}{|x_{F0}(0)|}\right). \quad (2.9b)$$

In the slab limit of $\alpha \to \infty$, equation (2.9a) with equation (2.9b) gives the well known behavior of the flux front in the slab sample:

$$|x_{F0}(0)| \to 1 - \frac{H_e}{J_C D_X}\ ;\ \alpha \to \infty. \quad (2.9c)$$

This fact may gives a support that equation (2.8) is a fairly good approximation for the expression for the flux front even for the case of $\alpha \neq 0$, so far as the shielding condition at the center of the cross section of the tape is concerned.

For the superconducting tape with $\alpha \ll 1$, the small quantities less than the order of magnitude of α^2 can be disregarded. If we retain up to the linear terms of α in equation (2.9a), the functional form of $\Delta = \Delta(\alpha,\ A)$ is determined by equating equation (2.9a) to equation (2.7a):

$$\ln\left(\frac{1+A\Delta(\alpha,\ A)}{|x_{F0}(0)|+\alpha A\Delta}\right) = \ln\left(\frac{1+A}{|x_{F0}(0)|}\right), \quad (2.10a)$$

and the result is given by

$$\Delta(\alpha,\ A) = 1 + \alpha\sqrt{\frac{1+A}{1-A}} = 1 + \alpha \exp(h_e). \quad (2.10b)$$

If equation (2.8) with equation (2.10b) were the exact solution of the critical state model even in the tape with $\alpha \neq 0$, then $\tilde{B}_J(0\ ;\ \alpha)$ given by equation (2.9a) with equation (2.9b) should satisfy the shielding condition given by $\tilde{B}(0\ ;\ \alpha) = \mu_0 H_e + \tilde{B}_J(0\ ;\ \alpha) = 0$.

In figure 2, the value of $\tilde{B}(0\ ;\ \alpha)/\mu_0 H_e$ for the shapes of the flux front, x_F, given by (2.8) with equation (2.10b) is plotted against α for $h_e = 0.05$ and 0.5. We can see that equation (2.8) with equation (2.10b) may be applicable to the derivation of the theoretical expression for the AC loss at least in the range of $\alpha < 0.3$, because in this range the value of $\tilde{B}(0\ ;\ \alpha)/\mu_0 H_e$ is less than 1/10 for the case of $\alpha < 0.3$. This conclusion is consistent with the fact that the terms of the order of α^2 were disregarded in the derivation of equation (2.10b).

2.2 Expression for the AC loss in the range of magnetic field of $H_m < H_p$

Let us consider the case that the AC magnetic field

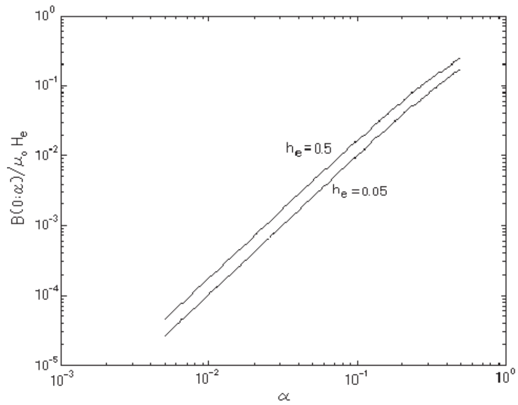

Fig. 2. α dependences of $\tilde{B}(0;\ \alpha)/\mu_0 H_e$ for $h_e = 0.05$ and 0.5

with the amplitude of H_m is applied to the concerning superconducting tape with $10^{-3} \ll \alpha \ll 1$ in the perpendicular direction, *i.e.*, in the Y-direction, where the variation speed of the AC field with time is assumed as low enough so that the critical state model is applicable to the derivation of the theoretical expression for the AC loss.

In this paper, let us confine the discussion to the case of $H_m < H_P$ for simplicity, where H_P is the centering field at which the flux front reaches the center of the cross section of the tape. The expression for the case of $H_m \geq H_P$ can also be derived along the similar scheme to that in the present section.

The magnetization in the initial increasing process of the applied magnetic field, $M_i(h_e)$ is defined by

$$M_i(h_e) \equiv -\frac{1}{2D_X D_Y}\int_0^1 D_Y \mathrm{d}y \int_{x_F(y)}^1 D_X \mathrm{d}x\, 2D_X x \mu_0 J_c(x, y). \qquad (2.11a)$$

For the present limiting case of $J_c(x, y) = \lambda J_c$, equation (2.11a) is reduced to

$$M_i(h_e) = -\frac{\mu_0 J_c}{2} D_X \int_0^1 \mathrm{d}y [1 - x_F(y)^2]. \qquad (2.11b)$$

By carrying out the above integration with the aid of equation (2.8), we get

$$M_i(h_e) = -\frac{1}{2}\mu_0 J_c D_X A \frac{A + \Delta(\alpha, A)}{1 + A\Delta(\alpha, A)}. \qquad (2.11c)$$

With the aid of equations (2.7d) and (2.10b), equation (2.11c) is reduced to

$$M_i(h_e) \simeq -\frac{1}{2}\mu_0 J_c D_X \tanh h_e [1 + \alpha \exp(-h_e)]. \qquad (2.11d)$$

In the following decreasing process of the applied magnetic field from $H_m \equiv h_m H_{pB}$ to $-H_m \equiv -h_m H_{pB}$, the current density of the induced current is given by λJ_c in the intermediate region between the inner flux front and the outer flux front, which is characterized by the range on the X axis $|x_{Fim}(0)| = \sqrt{1 - A^2(h_m)} < |x| < |x_{F\downarrow}(0)| = \sqrt{1 - A^2((h_m - h_e)/2)}$, and is given by $-\lambda J_c$ in the outside region of $|x_{F\downarrow}(0)| < |x| < 1$, as schematically shown in figure 1b.

Then the magnetization is given by

$$M_\downarrow(h_e) = M_i(h_m) - 2M_i\left(\frac{h_m - h_e}{2}\right). \qquad (2.12a)$$

Similarly, the magnetization in the following increasing process of the applied magnetic field, shown schematically in figure 1c, is given by

$$M_\uparrow(h_e) = -M_i(h_m) + 2M_i\left(\frac{h_m + h_e}{2}\right). \qquad (2.12b)$$

Then the AC loss per cycle per unit volume, W, is given by

$$W(h_m) = H_{pB}\int_{-h_m}^{+h_m} \mathrm{d}h_e [M_\downarrow(h_e) - M_\uparrow(h_e)],$$
$$= W_0(h_m) + \alpha W_1(h_m)\ ;\ h_m \equiv H_m / H_{pB}, \qquad (2.13a)$$

$$w_0(h_m) \equiv W_0(h_m)/(4/\pi)\mu_0 J_c^2 D_X D_Y$$
$$= 2\ln(\cosh h_m) - h_m \tanh h_m, \qquad (2.13b)$$

$$w_1(h_m) \equiv W_1(h_m)/(4/\pi)\mu_0 J_c^2 D_X D_Y$$
$$= 4\tan^{-1}(\tanh(h_m/2)) - 2[1 - \exp(-h_m)]$$
$$- h_m \tanh h_m \exp(-h_m). \qquad (2.13c)$$

Equation (2.13b) is the same as the expression for W obtained by Brandt and Indenbom [2] for the limit of $\alpha \to 0$.

It may be worth noticing that, for the case of $h_m \ll 1$, equations (2.13a)~(2.13c) are reduced to

$$W(h_m) \to \frac{4}{\pi}\mu_0 J_c^2 D_X D_Y \left(\frac{1}{6}h_m^4 + \frac{1}{3}\alpha h_m^3\right)\ ;\ h_m \ll 1. \qquad (2.14)$$

It is to be emphasized that the absolute value of the AC loss is decreased noticeably due to the decrease of the width, $D_X = D_Y/\alpha$. As can be seen from figure 3, however, the deviation of the normalized shape of the AC loss, $w(h_m) = w_0(h_m) + w_1(h_m)$, from $w_0(h_m)$ for $\alpha \sim 0$ becomes noticeable only at $\alpha > 5 \times 10^{-2}$.

On the other hand, the present expression for the AC loss given by equations (2.13a)~(2.13c) is expected to be a good approximation for $\alpha^2 < 10^{-1}$, as mentioned in Section 2.1.

Then the present expression for $W(h_m)$ is expected to

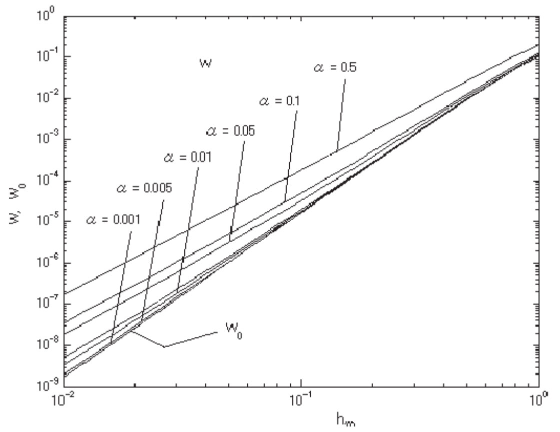

Fig. 3. h_m dependences of normalized loss density $w(h_m)$ for $\alpha = 0.001 \sim 0.5$

be useful for the practical purposes only for the range of $0.01 < \alpha < 0.3$, while the expression proposed by Brandt and Indenbom [2] can be applicable for the range of $\alpha < 0.01$.

3. Effects of the B-dependence of the critical current density on the AC loss

Let us consider the superconducting tape for which the B-dependence of the critical current density can be described well by

$$J_C(B) = J_C(0)\exp(-B/B_0), \tag{3.1}$$

where B_0 is a function of the temperature and also depends on the kind of material as well as the fabrication conditions of the tape. This type of the B-dependence of the critical current density appears in many samples of the high-T_C cuprate superconductors [9, 10]. It is to be noted that the limit of the constant critical current density discussed in the previous section can be formally obtained by taking the limit of $B_0 \to \infty$ and replacing $J_C(0)$ by J_C in equation (3.1).

For investigating the effects of the B-dependence of the critical current density on the AC loss, we have only to consider the film limit of $\alpha \to 0$, because the effect of the B-dependence of the critical current density on αW_1 can hardly be detected experimentally for the case of $10^{-3} \ll \alpha \ll 1$.

As the shape of the flux front in the presence of the B-dependence of the critical current density, therefore, let us adopt the same functional form as equation (2.8):

$$|x_{F\beta}(y)| = |x_{F0}(0)| \frac{\tan\xi(y)}{\sqrt{A^2[\Delta(B_0, A)]^2 + [\tan\xi(y)]^2}}, \tag{3.2a}$$

$$|x_{F0}(0)| = \sqrt{1-A^2} = \text{sech}\, h_e. \tag{3.2b}$$

Then the flux penetration of the flux front along the X-direction, $|x_{F0}(0)|$, is subject to the characteristic field, $H_{pB} = (2/\pi)J_C D_Y$ in the film with $\alpha \to 0$ as can be seen from equations (2.7c) with equations (2.7d) and (2.7b), whereas the penetration of the flux front is subject to $H_{pS} \equiv J_C D_X$ in the slab sample with $\alpha \to 0$ as can be seen from equation (2.9c).

In the film limit of $\alpha \sim 0$, therefore, the distribution of the flux density along the X-axis is described by the following Maxwell equation:

$$\frac{dB(x, y)}{dx} = \frac{2}{\pi}\mu_0 D_Y J_c(0)\exp\left(-\frac{B(x, y)}{B_0}\right); \quad x_{F\beta}(y) < x < 1. \tag{3.3a}$$

The solution of the above Maxwell equation is given by

$$\exp\left(-\frac{B(x, y)}{B_0}\right) = \frac{1}{1+\beta_0[x - x_{F\beta}(y)]}; \quad x_{F\beta}(y) < x < 1. \tag{3.3b}$$

In equation (3.3b), β_0 is defined by

$$\beta_0 \equiv \frac{(2/\pi)\mu_0 J_c(0) D_Y}{B_0}. \tag{3.4a}$$

It is to be noted that β_0 defined by equation (3.4a) is much smaller than unity:

$$\beta_0 \ll 1, \tag{3.4b}$$

because the value of D_Y is so small for the concerning tape with $\alpha \sim 0$ that the value of the numerator in equation (3.3a) is usually much smaller than the value of B_0 [8, 9].

Inserting equation (3.3b) into equation (3.1), we get

$$J_c(x,\ y)=J_c(B(x,\ y))=\frac{J_c(0)}{1+\beta_0[x-x_{F\beta}(y)]}. \quad (3.3c)$$

Inserting equation (3.3c) into equation (2.5b) and taking the limit of $\alpha\sim 0$, we get

$$\begin{aligned}\tilde{B}_J(0\ ;\ \beta_0)&=\mu_0 H_{pB}\int_0^1 \mathrm{d}y_0 \\ &\qquad\frac{\ln(x_{F\beta}(y_0))+\ln(1+\beta_0[1-x_{F\beta}(y_0)])}{1-\beta_0 x_{F\beta}(y_0)} \\ &\simeq \mu_0 H_{pB}\int_0^1 \mathrm{d}y_0\{\ln(x_{F\beta}(y_0))+\beta_0[1-x_{F0}(y_0)]\} \\ &=-\mu_0 H_{pB}\Big\{\ln\Big(\frac{1+A\,\Delta(B_0\ ;\ A)}{\sqrt{1-A^2}}\Big) \\ &\qquad -\beta_0\frac{2}{\pi}\tan^{-1}\Big(\frac{A}{\sqrt{1-A^2}}\Big)\Big\}. \quad (3.5a)\end{aligned}$$

The functional form of $\Delta(B_0\ ;\ A)$ introduced in equation (3.2a) can be determined by the same process as in the section 2.1. Since the value of $\Delta(B_0\ ;\ A)$ is reduced to 1 in the absence of the B-dependence of the critical current density, equation (3.5a) is reduced to

$$\tilde{B}_J(0\ ;\ \beta_0\to 0)=-\mu_0 H_{pB}\ln\Big(\frac{1+A}{\sqrt{1-A^2}}\Big), \quad (3.5b)$$

which is the same as $\tilde{B}_J(0\ ;\ \alpha\to 0)$ given by equation (2.7a).

By equating equation (3.5a) to equation (3.5b), we get

$$\begin{aligned}\Delta(B_0,\ A)&=1+\beta_0\frac{\exp(h_e)}{\sinh h_e}\frac{2}{\pi}\tan^{-1}(\sinh h_e) \\ &\simeq 1+\Big(\frac{2}{\pi}\beta_0\Big)\exp(h_e). \quad (3.6)\end{aligned}$$

The magnetization in the initial increasing process of the applied magnetic filed is given by equation (2.11a), where $J_c(x,\ y)$ is given by equation (3.3c). If we retain only the linear terms of β_0, the expression for the initial magnetization is given by

$$M_i(h_e)=M_{i0}(h_e)-\beta_0 M_{i2}(h_e)\ ; \quad (3.7a)$$

$$\begin{aligned}M_{i0}(h_e)&=-\frac{1}{2}\mu_0 J_c(0)D_X\int_0^1 \mathrm{d}y[1-x_{F\beta}(y)^2] \\ &=-\frac{1}{2}\mu_0 J_c(0)D_X A\frac{A+\Delta(B_0,\ A)}{1+A\,\Delta(B_0,\ A)}, \quad (3.7b)\end{aligned}$$

$$\begin{aligned}M_{i2}(h_e)&=-\frac{1}{2}\mu_0 J_c(0)D_X\int_0^1 \mathrm{d}y\Big\{[1-x_{F0}(y)] \\ &\qquad -\frac{1}{3}[1-x_{F0}(y)^3]\Big\} \\ &=-\frac{1}{2}\mu_0 J_c(0)D_X\frac{2}{3\pi}\Big[2\tan^{-1}\Big(\frac{A}{\sqrt{1-A^2}}\Big) \\ &\qquad -A\sqrt{1-A^2}\Big] \\ &=-\frac{1}{2}\mu_0 J_c(0)D_X\frac{2}{3\pi}[2\tan^{-1}(\sinh h_e) \\ &\qquad -\sinh h_e(\mathrm{sech}\,h_e)^2] \\ &\simeq -\frac{1}{2}\mu_0 J_c(0)D_X\frac{2}{3\pi}\sinh h_e[2-(\mathrm{sech}\,h_e)^2]. \quad (3.7c)\end{aligned}$$

Since equations (3.6) and (3.7b) are same as equations (2.10b) and (2.11c), respectively, if α is replaced by $(2/\pi)\beta_0$, we get the following expression for the AC loss :

$$W(h_m)=W_0(h_m)+(2/\pi)\beta_0[W_1(h_m)+W_2(h_m)], \quad (3.8a)$$

where $W_0(h_m)$ is given by replacing J_c by $J_c(0)$ in equation (2.13b), $W_1(h_m)$ is given by replacing J_c by $J_c(0)$ in equation (2.13c), and $W_2(h_m)$ is given by

$$\begin{aligned}W_2(h_m)&=\frac{8}{3\pi}\mu_0 J_c(0)^2 D_X D_Y \\ &\times\Big[h_m\sinh h_m\Big(1-\frac{1}{2}(\mathrm{sech}\,h_m)^2\Big) \\ &\qquad +(1-\mathrm{sech}\,h_m)-2(\cosh h_m-1)\Big]. \quad (3.8b)\end{aligned}$$

It may be worth noticing that, for $h_m\ll 1$, equations (3.8a) is reduced to

$$\begin{aligned}W(h_m)\to\frac{4}{\pi}\mu_0 J_c(0)^2 D_X D_Y\Big[\Big(\frac{1}{6}+\Big(\frac{2}{\pi}\beta_0\Big)\frac{7}{36}\Big)h_m^4 \\ +\Big(\frac{2}{\pi}\beta_0\Big)\frac{1}{3}h_m^3\Big]. \quad (3.8c)\end{aligned}$$

In figure 4, some examples of the present result are compared with the special case of disregarding the magnetic-flux-density dependence of the critical current density. As can be seen from figure 4, the effect of the magnetic-flux-density dependence given by equation (3.1) becomes noticeable for $\beta_0>5\times 10^{-2}$.

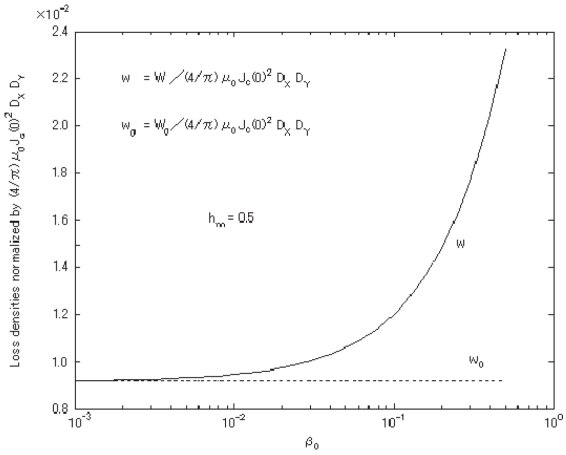

Fig. 4 β_0 dependence of normalized loss density w for $h_m=0.5$

4. Summary of the theoretical results

For the superconducting tape with the rectangular cross section characterized by the aspect ratio of $1/\alpha\equiv D_X/D_Y\gg1$ and also by the critical current density having the B-dependence given by

$$J_c(B)=J_c(0)\exp(-B/B_0),\tag{3.1}$$

the AC loss in the perpendicular AC field applied to the Y-direction with the amplitude of $H_m<H_P$ is given by

$$W(h_m)=W_0(h_m)+\left(\alpha+\frac{2}{\pi}\frac{\mu_0H_{pB}}{B_0}\right)W_1(h_m)+\frac{2}{\pi}\frac{\mu_0H_{pB}}{B_0}W_2(h_m)\ ;\tag{4.1a}$$

$$h_m\equiv H_m/H_{pB},\tag{4.1b}$$

where H_{pB} is defined by equation (2.7b).

In equation (4.1a), $W_0(h_m)$ is given by

$$W_0(h_m)=\frac{4}{\pi}\mu_0J_c(0)^2D_XD_Y[2\ln(\cosh h_m)-h_m\tanh h_m].\tag{4.1c}$$

Furthermore, $W_1(h_m)$ is given by

$$W_1(h_m)=\frac{4}{\pi}\mu_0J_c(0)^2D_XD_Y\times[4\tan^{-1}(\tanh(h_m/2))-2[1-\exp(-h_m)]-h_m\tanh h_m\exp(-h_m)]\tag{2.13c}$$

and $W_2(h_m)$ is given by

$$W_2(h_m)=\frac{8}{3\pi}\mu_0J_c(0)^2D_XD_Y\times\left[h_m\sinh h_m\left(1-\frac{1}{2}(\text{sech}\,h_m)^2\right)+(1-\text{sech}\,h_m)-2(\cosh h_m-1)\right].\tag{3.8b}$$

For $h_m\ll1$, equation (4.1a) is reduced to

$$W(h_m)\to\frac{4}{\pi}\mu_0J_c(0)^2D_XD_Y\left[\left(1+\frac{7}{3\pi}\frac{\mu_0H_{pB}}{B_0}\right)\frac{1}{6}h_m^4+\left(\alpha+\frac{2}{\pi}\frac{\mu_0H_{pB}}{B_0}\right)\frac{1}{3}h_m^3\right].\tag{4.2}$$

Since the terms of the order of α^2 and of $(\mu_0H_{pB}/B_0)^2$ are neglected in the derivation of the present expression, the present expression is expected to be a good approximation for $\alpha^2<1/10$ and $(\mu_0H_{pB}/B_0)^2<1/10$, while the expression proposed by Brandt and Indenbom [2] is applicable only for $\alpha<1/10^2$.

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Appendix

When the applied magnetic field is initially applied to the superconducting tape with a rectangular cross section, and is increased from $H_e=0$, the centering field, H_P, is defined as the applied magnetic field, at which the flux front reaches the center of the tape. Since the current density distribution at $H_e=H_P$ is given as $J=+J_c$ for $x>0$ and $J=-J_c$ for $x<0$, equation (2.5a) is reduced to [8]

$$\mu_0 H_P = -\tilde{B}_J(\tilde{z}) = \frac{\mu_0 J_c}{2\pi}\alpha D_X \int_{-1}^{+1} dy_0 \left[\int_0^1 dx_0 - \int_{-1}^0 dx_0\right] \times \frac{1}{x_0 + i\alpha y_0} \quad (A.1a)$$

$$= \frac{2}{\pi}\mu_0 J_c D_X\left[\tan^{-1}(\alpha) + \alpha \ln\sqrt{\frac{1+\alpha^2}{\alpha^2}}\right]. \quad (A.1b)$$

In the initial increasing process of H_e in the range of $0<H_e<H_P$, on the other hand, the region inside the flux front given by equation (2.4a) still remains. Let us investigate the values of $\tilde{B}_J(\tilde{z})$ in the region inside the flux front, defined by $0\leq \bar{x}\equiv|x|/|x_{F0}(0)|<1$. If we consider $\tilde{B}_J(\tilde{z})$ on the X axis by assuming that the shape of the flux front is given by equation (2.6a), then equation (2.5b) is reduced to

$$\tilde{B}_J(\bar{x},0) = -\frac{\mu_0 H_{pB}}{4}\int_0^1 dy_0 \ln\left(\frac{[1-|x_{F0}(0)|\bar{x}]^2+\alpha^2 y_0^2}{[x_F(y_0)-|x_{F0}(0)|\bar{x}]^2+\alpha^2 y_0^2} \times \frac{[1+|x_{F0}(0)|\bar{x}]^2+\alpha^2 y_0^2}{[x_F(y_0)+|x_{F0}(0)|\bar{x}]^2+\alpha^2 y_0^2}\right) - \frac{\mu_0 H_{pB}}{4}\int_0^1 dy_0 \ln\left(\frac{[(\tan\xi)^2+A^2]^2}{g(\xi,\ y_0)}\right);$$

$$\xi \equiv \frac{\pi}{2}(1-y_0). \quad (A.2c)$$

In equation (A.2c), $g(\xi,\ y_0)$ has the following functional form:

$$g(\xi,\ y_0) \equiv (\tan\xi)^4 + g_1(\alpha y_0)(\tan\xi)^2 + g_0(\alpha y_0). \quad (A.2d)$$

Unfortunately, the exact integration of the second integration in equation (A.2c) can hardly be carried out analytically. If we notice that the point of $\tan\xi=0$ is corresponding to the point of $y_0=1$, however, the second term in equation (A.2c) can be well approximated by replacing $g(\xi,\ y_0)$ by $g(\xi,\ 1)$. Then, the integrations in equation (A.2c) can be carried out analytically with the aid of the following integration formulae:

$$\int_0^1 dy_0 \ln\left(\frac{c_+^2+\alpha^2 y_0^2}{c_-^2+\alpha^2 y_0^2}\right) = \ln\left(\frac{c_+^2+\alpha^2}{c_-^2+\alpha^2}\right) + 2\left[\frac{c_+}{\alpha}\tan^{-1}\left(\frac{\alpha}{c_+}\right) - \frac{c_-}{\alpha}\tan^{-1}\left(\frac{\alpha}{c_-}\right)\right], \quad (A.3a)$$

$$\int_0^1 dy_0 \ln([(\tan\xi)^2+C_+^2][(\tan\xi)^2+C_-^2]) = 2\ln([1+C_+][1+C_-]). \quad (A.3b)$$

If we put $\bar{x}=0$, the resulting expression is obtained by putting $\Delta=1$ in equation (2.7b). The difference between the analytic result and the numerically calculated exact result is negligibly small, as can be seen in figure 2.

If we retain up to the linear terms of α, on the other hand, equation (A.2c) is reduced to

$$\tilde{B}_J(\bar{x},\ \alpha) = -\frac{1}{2}\mu_0 H_{pB}\ln\left(\frac{(1+A)^2[(1-\bar{x}^2)+\bar{x}^2A^2]}{|x_{F0}(0)|^2[(1-\bar{x}^2)+\bar{x}^2A^2]+2|x_{F0}(0)|\frac{A}{\sqrt{1-\bar{x}^2}}\alpha}\right). \quad (A.4)$$

It is to be emphasized that the right-hand-side of equation (A.4) becomes independent of $\bar{x}$ inside the flux front in the limit of $\alpha\to 0$, where $\bar{x}$ takes any value of $0\leq\bar{x}<1$. If we choose the relation of $|x_{F0}(0)|$ and H_e as $|x_{F0}(0)|=\text{sech}\, h_e$ given by equation (2.7c), therefore, the magnetic flux at any point on the X axis inside the flux front is completely shielded to be zero for the case of the film limit of $\alpha\to 0$. This is the reflection of the fact [2, 6] that equation (2.6a) is almost the exact solution of the critical state model in the film limit [2, 6].

It is, however, to be noted that equation (2.6a) can hardly be regarded as the exact solution even for the limit of $\alpha\to 0$ from the following 2 points:

First, at least the value of $d^2\tilde{B}_J(\tilde{z})/d\tilde{z}^2$ with the definition of $\tilde{z}\equiv x+iy$ should be zero in the region inside the flux front, if equation (2.6a) is the exact solution. As can be easily confirmed, however, the above value in the limit of $\alpha\to 0$ is not zero.

Secondly, according to equation (2.6a), the value of the flux front at $x=0$ is always fixed to $y=1$, whereas the result of the numerical calculation [8] indicates that the value of the flux front at $x=0$ begins to depart from $y=1$ at $H_e \sim H_P$ and approaches to $y=0$ as $H_e \sim H_P$. Nevertheless, we may conclude that equation (2.6a) is a very good solution, except for only in the very vicinity of $H_e \sim H_P$.