The Roads to Incentive Compatibility: Mechanism Games vs Catalog Games

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Abstract

We construct a principal-agent game involving network formation over layered networks with asymmetric information. Introducing a novel approach called delegated networking principle, we aim to address two questions: (1) Is it possible for the principal to design a mechanism to incentivize the agents to truthfully disclose their private information and adhere to the recommended connections by the principal? (2) Is it possible for the principal to attain the same outcome as that achieved via the centralized mechanism by selecting a profile of allowable connection sets and delegating connection choices to each pair of players?

Keywords: Incentive Compatible Networks, Delegated Networking Principle, Bilateral Incentive Compatibility, Mechanism Design, Catalog Games

1 Introduction

We consider the problem faced by a principal who seeks to the structure incentives faced by a set of agents in forming a network of connections among themselves in such a way that each agent, in light of his private information, forms connections that are the best interest of the principal. Thus, the principal seeks to influence - if not control - not only who is interacting (i.e., which pairs of agents form connections) but also how they are interacting. In many networking situations, however, the principal, in addition to not being able to observe who is interacting and how, does not have complete information concerning the agent's "type" (i.e., a parameter summarizing the agent's basic characteristics). Thus, there is an adverse selection problem.

To address the issues raised above, we construct a principal-agent game of network formation (over layered networks) with asymmetric information and introduce a novel approach called delegated networking. We show, under relatively mild conditions on our game-theoretic model of network formation, that strategic network formation with incomplete information, implemented via a mechanism and centralized reporting, is equivalent to implementation via delegated networking with monitoring. Thus, we show that the delegation principle of contracting theory holds for games of network formation with incomplete information.

Because the basic strategic ingredients of our game of network formation are bilateral connections, it is useful to view each agent pair, ij, as a player or a node with a player's club choice representing the resolution of the "whether or not to connect" and "how to connect" part of the problem.

2 Network Formation Under Incomplete Information

2.1 Primitives

Assume the following:

(1) N is a finite set of agents, consisting of n agents, equipped with the discrete metric η_N, having typical elements i and j.

(2) $N^2 := N \times N$ is the set of agent pairs, consisting of n^2 pairs, each agent pair representing a player, equipped with the discrete metric $\eta_{N\times N} := \eta_N + \eta_N$, having typical elements ij (including the diagonal pairs $ii \in N^2$).

(3) C is a finite set of clubs equipped with discrete metric η_C having typical element c, containing a special "no interaction" club c_0.

(4) $(S, \mathcal{B}(S))$ is a space consisting of mutually observable states, $s \in S$, where S is a complete, separable metric (Polish) space with metric ρ_S and Borel σ-field $\mathcal{B}(S)$.

(5) $(T_i, \mathcal{B}(T_i))$ is a space consisting of ith agent types, $t_i \in T_i$, where T_i is a complete, separable metric (Polish) space with metric ρ_{T_i} and Borel σ-field $\mathcal{B}(T_i)$.

(6) $T_{ij} := T_i \times T_j$ is the space of player ij's possible types, $t_{ij} := (t_i, t_j) \in T_{ij}$, equipped with the Borel product σ-field, $\mathcal{B}(T_{ij}) := \mathcal{B}(T_i) \times \mathcal{B}(T_j)$.

(7) $T = \Pi_i T_i$ is the space of agent type profiles (n-tuples), $t \in T$, equipped with the Borel product σ -field, $\mathcal{B}(T) = \Pi_i \mathcal{B}(T_i)$.

(8) A_c is a convex, compact metrizable subset of a locally convex Hausdorff topological vector space E_c containing all possible category c arc types, a_c, equipped with a metric ρ_c compatible with the locally convex topology inherited from E_c.

(9) $A(\cdot, c)$ is club $c's$ feasible arc correspondence, a set-valued mapping from the set of all players, ij, taking values in the collection, 2^{A_c}, of ρ_c -closed subsets of A_c such that for each player, ij,

$$A(ijc) \subset A_c \subset E_c.$$

Alternatively, the correspondence, $A(ij\cdot)$, is player $ij's$ feasible arc correspondence across clubs, i.e., $A(ijc) \in 2^{A_c}$.

We will refer to our list of primitives together with our assumptions as **[A-1]**(γ), $\gamma = 1, 2, \ldots, 9$.

2.2 Club Networks

In our club network model, we will assume that each player, $ij \in N^2$, can join multiple clubs, $c \in C$, and in each club player ij takes a particular action a from a feasible set of actions, $A(ijc)$, relevant to that club. This set of relevant actions for each player-club pair is given by the feasible arc correspondence, $ijc \longrightarrow A(ijc)$. We have the following formal definition of a club network.

Definition 1 (Club Networks). Given arc sets $\{A_c : c \in C\}$, with $A := \cup_c A_c$, node set, $N^2 \cup C$, and feasible arc correspondence, $ijc \longrightarrow A(ijc)$, a club network is a nonempty, closed subset, G, of $A \times \left(N^2 \times C\right)$ such that (i) $|G(ijc)| \leq 1$ and $|G(ijc)| = 1$ for some $c \in C$, (ii) if for $c \in C$, $|G(ijc)| = 1$, then $(a, (ij, c)) \in G$ if and only if $a \in A(ijc)$, and (iii) for all $ij \in N^2$,

$$|G(ijc_0)| = 1 \text{ iff } |G(ijc)| = 0 \text{ for all } c \in C\backslash\{c_0\}.$$

We will denote by $\mathbb{G}$ the collection of all feasible club networks. Thus,

$$\mathbb{G} := \left\{ \begin{array}{l} G \in P_f(A \times (N^2 \times C)) : \\ \text{satisfying } (i), (ii), \text{and } (iii) \end{array} \right\}.$$

Thus, in a club network a typical connection is given by

$$(a, (ij, c)) \in A \times ((N \times N) \times C),$$

where connection, $(a, (ij, c))$, indicates that player ij is in club c and that in this club player ij takes feasible action $a \in A(ijc)$.

Then, we will assume the following concerning each player's payoff function, $u_{ij}(\cdot, \cdot)$ (see Balder (1997), and Bloch and Jackson (2007)).

[A-2] (Players' payoff functions are Caratheodory and additively coupled)
We will assume that for each $(\omega, G) \in \Omega \times \mathbb{G}$,

$$u_{ij}(\omega, G) = v_{ij}(\omega_{ij}, G_{ij}) + \textstyle\sum_{i'j' \neq ij} \bar{v}_{ij}(\omega_{ij}, \omega_{i'j'}, G_{i'j'}),$$

for some functions $v, \bar{v}$, where v is Caratheodory.

3 Mechanism Games vs Catalog Games

3.1 Mechanism Games

In this section, we analyze the network formation problem under incomplete information as a principal-agent network formation game with adverse selection, assuming that the principal is allowed to design a profile of network formation mechanisms, so as to induce players to reveal their types and to follow the connection recommendations of the mechanism.

3.2 Catalog Games

A catalog is a closed set of feasible networks offered to players who choose the optimal network from the catalog, while the principal observes these choices to form recommendations. The Delegated Networking Principle characterizes all incentive-compatible mechanisms via catalogs of networks. This simplifies the principal-agent game, converting it into an equivalent unconstrained game over network catalogs.

Theorem 1 (The Delegated Networking Principle). Suppose assumptions [A-1] and [A-2] hold. The following statements are equivalent.

(1) $M_{ij}(\cdot) \in \mathbb{G}(\Omega_{ij}, \mathbb{G}_{ij})$, is incentive compatible, that is, for all ω_{ij} and ω'_{ij}

$$U_{ij}(\omega_{ij}, M_{ij}(\omega_{ij})) \geq U_{ij}(\omega_{ij}, M_{ij}(\omega'_{ij})).$$

(2) $M_{ij}(\cdot) \in \mathbb{M}(\Omega_{ij}, \mathbb{G}_{ij})$ is such that there exists a unique, minimal (by set inclusion) ij-catalog, $\mathcal{G}_{ij} \in P_{h_K f}(\mathbb{G}_{ij})$ satisfying

$$M_{ij}(\omega_{ij}) \in \Phi_{U_{ij}}(\omega_{ij}, \mathcal{G}_{ij}) \text{ for all } \omega_{ij}.$$

We prove the equivalence between mechanism and catalog game theoretic approaches to network formation under incomplete information. The Delegated Networking Principle offers a powerful tool for designing incentive-compatible networks, emphasizing the role of delegation in managing complex network interactions.

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